# Solutions To Additional Problems In "Noise Control: From Concept To Application. Second Edition"

by

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# 1

# Solutions to Additional Problems in Chapter 1

## Problem 1

See Section 1.2.

- Indirect source control.
- Direct source control.
- Transmission path control.
- Receiver control.

## Problem 2

(a) From Table 1.2, it can be seen that the 250 Hz octave band extends from 176 to 353 Hz = 177 bins.
The octave band level is then $55 + 10\log_{10}(177) = 77.5$ (dB re 20 µPa).

(b) Using Table 2.4, the 250 Hz octave band A-weighted SPL = $77.5 - 8.6 = 68.9$ (dBA re 20 µPa).

(c) Band number $= 10\log_{10} 250 = 24$.

## Problem 3

(a) From Equation (1.62), $\hat{p} = \sqrt{2} \times p_{ref} \times 10^{103/20} = 1.414 \times 2 \times 10^{-5} \times 10^{103/20} = 3.99$ (Pa)

(b) From Equation (1.62), $p_1 = p_{ref} 10^{105/20}$ and $p_2 = p_{ref} 10^{107/20}$

From Equation (1.72)
$p_t^2 = p_{ref}^2 \left(10^{10.5} + 10^{10.7} + 2 \times 10^{5.25} \times 10^{5.35} \times \cos\theta\right) = p_{ref}^2 \left(10^{10.3}\right)$ (Pa$^2$).

Thus, $\cos\theta = \left(\dfrac{10^{10.3} - 10^{10.5} - 10^{10.7}}{2 \times 10^{5.25} \times 10^{5.35}}\right) = -0.776$

and so $\theta = 141°$.

## Problem 4

(a) The noise levels due to the machine only for each of the three 1/3 octave bands can be calculated following Example 1.41 as:
$10\log_{10}\left(10^{98/10} - 10^{95/10}\right) = 95.0$ (dB re 20 µPa),
$10\log_{10}\left(10^{102/10} - 10^{98/10}\right) = 99.8$ (dB re 20 µPa),
$10\log_{10}\left(10^{96/10} - 10^{94/10}\right) = 91.7$ (dB re 20 µPa).
Using Equation (1.74), the $L_{eq}$ for the 500 Hz octave band is:
$10\log_{10}\left(10^{95/10} + 10^{99.8/10} + 10^{91.7/10}\right) = 101.5$ (dB re 20 µPa).

(b) The decibel reduction for the octave band is the average reduction over the three 1/3-octave bands making up the octave band. Thus:
dB reduction $= -10\log_{10}\frac{1}{3}\left[10^{-15/10} + 10^{-20/10} + 10^{-23/10}\right] = 18.1$ (dB).

## Problem 5

From equation (1.1), $c = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.35 \times 8.314 \times 1783}{0.0361}} = 745$ (m/s).

$$c_{\min} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.345 \times 8.314 \times 1782.5}{0.03605}} = 744 \text{ (m/s)}.$$

$$c_{\max} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.355 \times 8.314 \times 1783.5}{0.03605}} = 747 \text{ (m/s)}.$$

From equation (1.1), $\rho_{\max} = \frac{\gamma P}{c^2} = \frac{1.355 \times 101.4 \times 10^3}{743.6^2} = 0.248$ (kg/m$^3$).

$$\rho = \frac{\gamma P}{c^2} = \frac{1.35 \times 101.4 \times 10^3}{744.6^2} = 0.247 \text{ kg/m}^3.$$

$$\rho_{\min} = \frac{\gamma P}{c^2} = \frac{1.345 \times 101.4 \times 10^3}{746.5^2} = 0.246 \text{ (kg/m}^3\text{)}.$$

Surface area, $S = (\pi \times 4^2/4) \times 2 + \pi \times 4 \times 12 = 56\pi = 176$ (m$^2$).

Maximum $S = (\pi \times 4.05^2/4) \times 2 + \pi \times 4.05 \times 12.05 = 179$ (m$^2$).

Minimum $S = (\pi \times 3.95^2/4) \times 2 + \pi \times 3.95 \times 11.95 = 173$ (m$^2$).

Rearranging Equation (6.12), taking logs and also taking into account the different reference values used for sound pressure and sound power and multiplying each resulting term by 10 to obtain decibels, results in the following expression for sound power level.

$$L_W = L_p - 10\log_{10}\left(\frac{4(1-\bar{\alpha})}{S\bar{\alpha}}\right) - 10\log_{10}\left(\frac{\rho c}{400}\right)$$
$$= 112.5 - 10\log_{10}\left(\frac{4 \times 0.918}{176 \times 0.082}\right) - 10\log_{10}\left(\frac{745 \times 0.247}{400}\right) = 121.8 \text{ (dB re } 10^{-12}\text{ W)}.$$

To prove that this is the correct number of significant figures we need to look at maximum and minimum values.

Minimum value: $L_W = 112.55 - 10\log_{10}\left(\frac{4 \times 0.9185}{173 \times 0.0815}\right) - 10\log_{10}\left(\frac{747 \times 0.248}{400}\right)$
$= 121.73$ (dB re $10^{-12}$ W).

Maximum value: $L_W = 112.55 - 10\log_{10}\left(\frac{4 \times 0.9175}{179 \times 0.0825}\right) - 10\log_{10}\left(\frac{744 \times 0.246}{400}\right)$

$= 121.99$ (dB re $10^{-12}$ W).

So 1 decimal point is too accurate - the result should be given as 122 dB.

## Problem 6

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(a) From Equation (1.62), $p_1 = p_{ref} 10^{98.2/20}$ and $p_2 = p_{ref} 10^{96.1/20}$.

From Equation (1.72), $p_t^2 = p_{ref}^2 \left(10^{9.82} + 10^{9.61} + 2 \times 10^{4.91} \times 10^{4.805} \times \cos\theta\right)$ (Pa$^2$).

The quantity $\cos\theta$ varies between -1 and 1. So the maximum possible sound pressure level is:

$$L_{p(\max)} = 10 \log_{10} \left(\frac{p_t^2}{p_{ref}^2}\right) = 10 \log_{10} \left(10^{9.82} + 10^{9.61} + 2 \times 10^{4.91} \times 10^{4.805}\right)$$
$$= 103.2 \text{ (dB re } 20\,\mu\text{Pa).}$$

and the minimum possible sound pressure level is:

$$L_{p(\min)} = 10 \log_{10} \left(\frac{p_t^2}{p_{ref}^2}\right) = 10 \log_{10} \left(10^{9.82} + 10^{9.61} - 2 \times 10^{4.91} \times 10^{4.805}\right)$$
$$= 84.8 \text{ (dB re } 20\,\mu\text{Pa).}$$

(b) $L_p = 10 \log_{10} \left(10^{98.2/10} + 10^{96.1/10}\right) = 100.3$ (dB re 20 µPa).

## Problem 7

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Overall level = 85 (dB re 20 µPa).
From Table 1.2, it can be seen that the number of 1 Hz bands in the 1000 Hz 1/3 octave band is 1130 - 880 = 250.
Spectral density $= 85 - 10 \log_{10} 250 = 61.0$ (dB/Hz).

## Problem 8

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Using Equation (1.75), $\text{NR} = 10 \log_{10} \left[10^{-0/10} + 10^{-2/10}\right] - 10 \left[10^{-5/10} + 10^{-7/10} + 10^{-7/10} + 10^{-9/10}\right]$

$= 2.124 + 0.751 = 2.9$ (dB).

## Problem 9

(a) From equation (1.1), $c = \sqrt{\gamma RT/M} = \sqrt{1.35 \times 8.314 \times 363.2/0.038} = 327.5$ (m/s).

(b) $f = \dfrac{1000 \times 6}{60} = 100$ (Hz).

(c) Resonance - open ended organ pipe (see Equation (1.40)).

$f = \dfrac{Nc}{2L}; \quad L = \dfrac{Nc}{2f} = \dfrac{327.5}{200} = 1.64$ (m)

Or, if pipe assumed closed at one end (recip compressor end only - see Equation (1.39)).

$L = \dfrac{(2n-1)c}{4f} = \dfrac{327.5}{400} = 0.82$ (m).

(d) Assumptions:

(i) Temperature of gas is uniform in pipe.

(ii) Ends are effectively open.

## Problem 10

(a) The acoustic particle velocity may be written as $u(x,t) = A\mathrm{e}^{\mathrm{j}(\omega t - kx)}$. Using Equations (1.2) and (1.3), the acoustic velocity potential may be written as:

$$\phi(x,t) = -\int u\mathrm{d}x = \frac{A}{\mathrm{j}k}\mathrm{e}^{\mathrm{j}(\omega t - kx)}$$

and the acoustic pressure may be written as:

$$p(x,t) = \rho\frac{\partial \phi}{\partial t} = \rho A\frac{\mathrm{j}\omega}{\mathrm{j}k}\mathrm{e}^{\mathrm{j}(\omega t - kx)} = \rho c u$$

(b) Particle velocity is the magnitude of the motion of the particles disturbed during the passage of an acoustic wave, whereas the speed of sound refers to the speed at which the disturbance propagates. Acoustic particle velocity is a function of the loudness of the noise, whereas the speed of sound is independent of loudness.

(c) The specific acoustic impedance is the ratio of acoustic pressure to particle velocity. Using the preceding equations we obtain:

$$Z = \frac{p(x,t)}{u(x,t)} = \rho c\frac{A\mathrm{e}^{\mathrm{j}(\omega t - kx)}}{A\mathrm{e}^{\mathrm{j}(\omega t - kx)}} = \rho c$$

(d) To simplify the algebra, set the origin of the coordinate system at the rigid end of the tube as shown in Figure 1.1.

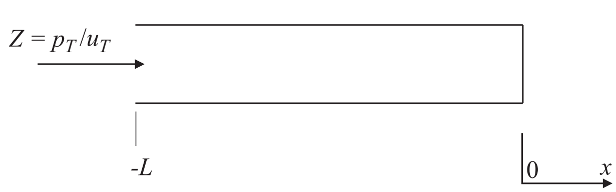


**FIGURE 1.1** Arrangement for Problem 11.

As the tube is terminated with an open end and the wavelength is large compared to the tube diameter, and the pressure at the end is zero, the reflected wave at the open end must have the same amplitude and opposite phase to the incident wave (ie a phase shift of 180° ). As the origin, $x = 0$ is at the point of reflection, the phase of the two waves must be 180° apart when $x = 0$. Of course if the origin were elsewhere, this would not be true and the following expressions would have to include an additional term (equal to the distance from the origin to the point of reflection) in the exponent of the reflected wave. With the origin at the point of reflection, the total acoustic pressure and particle velocity at any point in the tube may be written as:

$$p_T = A\left(\mathrm{e}^{\mathrm{j}(\omega t - kx)} - \mathrm{e}^{\mathrm{j}(\omega t + kx)}\right)$$

and

$$u_T = \frac{A}{\rho c}\left(\mathrm{e}^{\mathrm{j}(\omega t - kx)} + \mathrm{e}^{\mathrm{j}(\omega t + kx)}\right)$$

The specific acoustic impedance is then:

$$\frac{Z}{\rho c} = \frac{p}{u} = \frac{\left(\mathrm{e}^{-\mathrm{j}kx} - \mathrm{e}^{\mathrm{j}kx}\right)}{\left(\mathrm{e}^{-\mathrm{j}kx} + \mathrm{e}^{\mathrm{j}kx}\right)} = \frac{-\sinh(kx)}{\cosh(kx)} = -\mathrm{j}\tan(kx)$$

## Problem 11

(a) The spherical wave solution to the wave equation is given in the question as:

$$p = \frac{A}{r}\mathrm{e}^{\mathrm{j}(\omega t - kr)}$$

Using Equation (1.3) in the text, the velocity potential is:

$$\phi = \frac{A}{\mathrm{j}\rho r\omega}\mathrm{e}^{\mathrm{j}(\omega t - kr)}$$

Using the one dimensional form of Equation (1.2) in the text, the particle velocity is:

$$u = -\frac{\partial \phi}{\partial r} = \frac{\mathrm{j}kA}{\mathrm{j}r\omega\rho}\mathrm{e}^{\mathrm{j}(\omega t - kr)} + \frac{A}{\mathrm{j}r^2\omega\rho}\mathrm{e}^{\mathrm{j}(\omega t - kr)}$$

$$= \frac{A}{r\rho c}\mathrm{e}^{\mathrm{j}(\omega t - kr)}\left(1 - \frac{\mathrm{j}}{kr}\right) = \frac{p}{\rho c}\left(1 - \frac{\mathrm{j}}{kr}\right)$$

(b) Specific acoustic impedance, $Z = p/u$. Thus:

$$Z = \rho c\left(1 - \frac{\mathrm{j}}{kr}\right)^{-1} = \rho c\frac{\mathrm{j}kr}{1 + \mathrm{j}kr}$$

(c) The modulus of the specific acoustic impedance of the spherical wave is half that of a plane wave ($\rho c$) when

$$\left(\sqrt{1 + \frac{1}{(kr)^2}}\right)^{-1} = 0.5 \quad \text{or} \quad (kr)^2 = 0.333.$$

Thus, $r = (\lambda/2\pi)\sqrt{0.3333} = 0.092\lambda$.

## Problem 12

(a) Energy density is a scalar quantity which is directly related to the sound pressure level. It is a measure of the energy in an enclosed space. Sound intensity is a vector quantity and as such is not directly related to the sound pressure in an enclosed space. It is a measure of the energy propagating in a particular direction.

(b) In a diffuse field, energy propagation in any direction is equally likely. As sound intensity is a vector quantity, energy coming from two opposite directions will be characterised by intensities of opposite sign which cancel when they interact if they are the same magnitude and so in a diffuse field, the net result is zero total intensity.

## Problem 13

Using Equation (1.1), it can be shown that

$$\rho c = \frac{\gamma P_s}{\sqrt{\gamma RT/M}} = \frac{1.4 \times 101400}{\sqrt{1.4 \times 8.314 \times 313.2/0.029}} = 400.4 \text{ (kg m}^{-2}\text{ s}^{-1}\text{)}$$

## Problem 14

$\omega = 200\pi$, $\rho = 1.206$, $U_0 = 2$, $r_0 = 0.05$.

From Equation (1.61), power, $W = IS = (p^2/\rho c)4\pi r^2 = \dfrac{\hat{p}^2}{2\rho c}4\pi r^2 = \dfrac{|B|^2}{2\rho c r^2}4\pi r^2$ and

$|B|^2 = \omega^2 r_0^4 \rho^2 \hat{U}^2$.

Thus, $W = 2\pi\omega^2 r_0^4 \rho \hat{U}^2/c = 2\pi(2\pi \times 100)^2 \times 0.05^4 \times 1.205 \times 2^2/344 = 0.22$ (W).

## Problem 15

(a) From Equation (1.25), the higher order mode cut-on frequency is: $f_{co} = 0.586c/d$, where $d$ is the tube diameter.
Thus $f_{co} = 0.586 \times 343/0.05 = 4020$ (Hz).
Frequency range for plane waves = 0 to 4020 (Hz).

(b) From Equation (1.61), for plane waves, the acoustic power is:
$W = IS = \dfrac{S\langle p^2\rangle}{\rho c} = \rho c S\langle u^2\rangle$, where $S$ is the duct cross sectional area.
$\langle u^2\rangle = \omega^2\hat{\xi}^2/2 = (2\pi\times 500\times 0.0001)^2/2 = 0.04935$ (m/s)$^2$, where $\hat{\xi}$ is the displacement amplitude, related to the velocity amplitude by $\hat{u} = \hat{\xi}\omega$.
$\rho c = 1.206 \times 343 = 413.7$, $S = (\pi/4) \times 0.05^2$.
Thus, $W = 413.7 \times 1.964 \times 10^{-3} \times 0.04935 = 0.04$ (W).

(c) As power is proportional to the square of the cone velocity, the cone velocity squared should be kept constant which means that the displacement of the speaker cone should vary inversely with frequency.

# 2

# Solutions to Additional Problems in Chapter 2

## Problem 1

(a) Replacing the "8" with the actual number ("7") of hours over which the sound pressure level is to be averaged in Equation (2.6) gives:
$L_{Aeq} = 10\log_{10}\left\{\frac{1}{7}\left[1\times10^{9.7}+4\times10^{9.4}+2\times10^{9.2}\right]\right\} = 94.2$ (dBA re 20 µPa).

(b) From Equation (2.6), $L_{Aeq,8\text{h}} = 10\log_{10}\left\{\frac{1}{8}\left[1\times10^{9.7}+4\times10^{9.4}+2\times10^{9.2}\right]\right\}$
$= 93.6$ (dBA re 20 µPa).

(c) From Equation (2.10), $E_{A,8h} = 4\times8\times10^{(93.6-100)/10} = 7.3$ ($\text{Pa}^2\cdot\text{h}$).

## Problem 2

(a) From Equation (1.74) (including the octave band centre frequency A-weighting corrections of Table 2.4), the A-weighted level is:
$$10\log_{10}\left[10^{(46-26.2)/10}+10^{(41-16.1)/10}+10^{(38-8.6)/10}+10^{(40-3.2)/10}\right.$$
$$\left.+10^{43/10}+10^{(46+1.2)/10}+10^{(49+1.0)/10}+10^{50-1.1)/10}\right]$$
= 54.1 dBA (corresponding to A-weighted corrections at band centre frequency).

(b) NCB = (40 + 43 + 46 + 49)/4 = 45 (see Section 2.9.1.3).

(c) Spectrum is plotted on NCB curve (see Figure 2.10) in the following figure to determine if hissy or not.

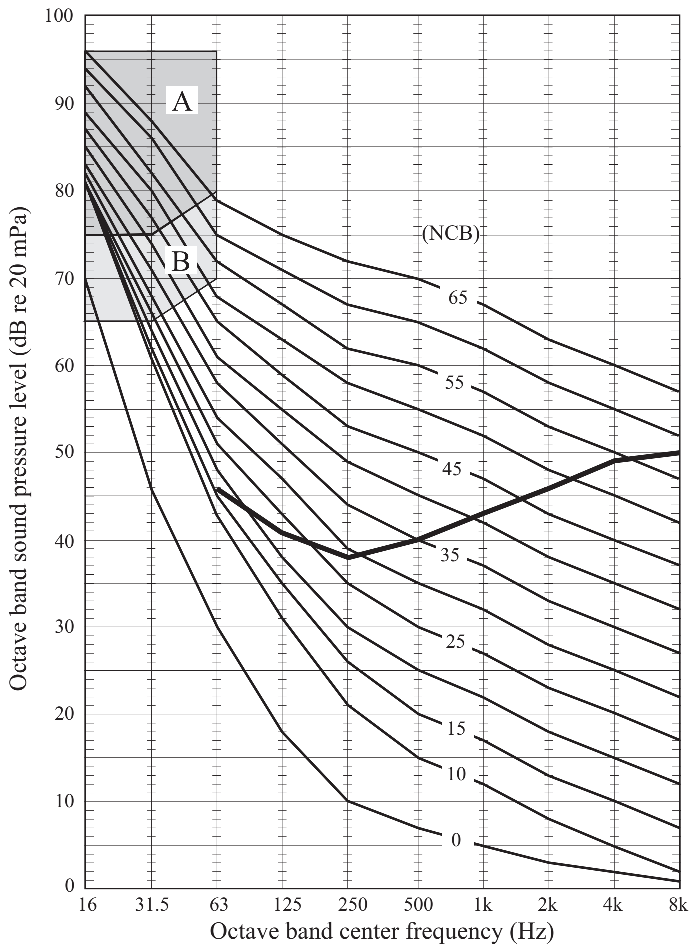


**FIGURE 2.1** NCB plot for Problem 2.

Best fit of NCB curve between 125 and 500 Hz is NCB = 29. Clearly band levels in 1000 Hz and higher exceed this so noise is hissy. No values in the bands 500 Hz and below exceed the NCB = 45 curve by more than 3dB so noise is not rumbly.

## Problem 3

(a) From Equation (1.74) (including the octave band centre frequency A-weighting corrections of Table 2.4), the A-weighted SPL is given by:

$L_{pA} = 10\log_{10}(10^{(9.5-0.86)} + 10^{(9.7-0.32)} + 10^{9.9}) = 100$ (dBA re 20 µPa).

(b) From Equation (2.35), with $L_B = 85$ and $L = 3$, the allowed daily exposure time in Australia is: $T_a = 8 \times 2^{-(100-85)/3} = 0.25$ (hours).

(c) From Equation (2.35), with $L_B = 90$ and $L = 5$, the allowed daily exposure time in USA is: $T_a = 8 \times 2^{-(100-90)/5} = 2$ (hours).

## Problem 4

(a) From Equation (2.6), $L_{Aeq,8h} = 10\log_{10}\frac{1}{8}\left[10^{9.1}\times 3 + 10^{9.5}\times 2 + 10^{10}\times 1\right]$
$= 94.0$ (dBA re 20 µPa).

(b) From Equation (2.35), with $L_B = 87$ and $L = 3$, the allowed daily exposure time in Europe is: $T_a = 8 \times 2^{-(94.0-87)/3} =$ 1.6 (hours) (or 1.2 hours to the 6 hour environment).
Reduction needed is 6.4 hours, so % reduction needed is $(6.4/8) \times 100 = 80\%$ or $(4.8/6) \times 100 = 80\%$.

## Problem 5

**TABLE 2.1** Data for Problem 5

| | Octave band centre frequency (Hz) | | | |
|---|---|---|---|---|
| | 63 | 125 | 500 | 1000 |
| $L_{eq,8h}$ (dB re 20 µPa) | 103 | 90 | 85 | 82 |
| A-weight correction (dB) | -26.2 | -16.1 | -3.2 | 0 |
| $L_{Aeq,8h}$ (dB re 20 µPa) | 76.8 | 73.9 | 81.8 | 82 |

(a) From Equation (2.6): Overall $L_{Aeq,8h} = 10\log_{10}\left(10^{76.8/10} + 10^{73.9/10} + 10^{81.8/10} + 10^{82}\right)$
$= 85.8$ (dBA re 20 µPa).

(b) Allowable exposure time, $T_a = 8 \times 2^{-(L_{Aeq,8h}-85)/3} = 8 \times 2^{-(85.82-85)/3} = 6.6$ (hours).

## Problem 6

See Section 1.2.4

(a) Measure the octave band (possibly 1/3 octave band) noise levels at all worker locations.

(b) Compare with allowable levels and determine required noise reductions having regard to the number of people affected at each location

(c) If excessive, measure the sound power levels of the noisy machines or get the levels from machine specs.

(d) Develop a computer model for predicting the noise level at each worker location and rank order the noise sources contribution to each location.

(e) Determine the most cost effective means of achieving the required environment by doing "what ifs" with the computer model. This will identify the most important items of equipment to be controlled.

(f) Look first at controlling the noise at the source. Look at the feasibility of both direct and indirect source control. List some possibilities.

(g) Look at what could be done by path control - again look at a number of possibilities including enclosures and barriers and mufflers.

(h) Look at what could be done to the workers such as personnel enclosures and hearing protection.

(i) Finally look at administrative controls such as moving equipment and personnel around.

## Problem 7

(a) From Equations (2.4) and (2.6),
$L_{Aeq} = 10\log_{10}\frac{1}{6}\left[10^{9.6}\times 1 + 10^{9.8}\times 0.5 + 10^{10}\times 1.5 + 10^{8.5}\times 3\right]$
$= 95.9$ (dB re 20 µPa).

(b) From Equation (2.6),
$L_{Aeq,8h} = 10\log_{10}\frac{1}{8}\left[10^{9.6}\times 1 + 10^{9.8}\times 0.5 + 10^{10}\times 1.5 + 10^{8.5}\times 3\right]$
$= 94.6$ (dB re 20 µPa).

(c) From Equation (2.35), with $L_B = 90$ and $L = 3$,
$T_A = 6\times 2^{-(94.6-90)/3} = 2.1$ (hours).

## Problem 8

According to Section 2.9.1.4, the RC value = (43 + 42 + 39)/3 = 41 (nearest integer). Plotting the octave band levels, we can see that below 500 Hz the values do not exceed the RC41 curve by more than 5 dB so the noise is not rumbly. The 4 kHz band exceeds the curve by more than 3 dB so the noise will be "hissy".

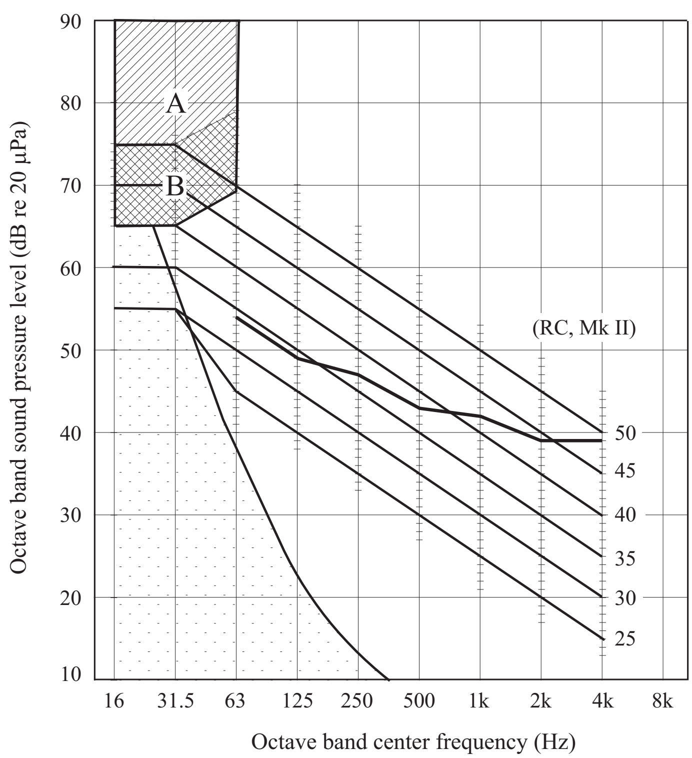


**FIGURE 2.2** RC plot for Problem 8.

## Problem 9

$p = (t^2 + 8t + 4) \times 10^{-2}$,

thus $p^2 = (t^4 + 16t^3 + 72t^2 + 64t + 16) \times 10^{-4}$

Average $p^2 = \langle p^2 \rangle = \frac{1}{8}\int_0^8 (t^4 + 16t^3 + 72t^2 + 64t + 16) \times 10^{-4} \mathrm{d}t$

$= \frac{1}{8}\left(\frac{t^5}{5} + \frac{16t^4}{4} + \frac{72t^3}{3} + \frac{64t^2}{2} + 16t\right) \times 10^{-4}$

$= \frac{1}{8}\left(\frac{8^5}{5} + 4 \times 8^4 + 24 \times 8^3 + 32 \times 8^2 + 16 \times 8\right) \times 10^{-4} = 0.4675\ (\mathrm{Pa}^2)$.

Using Equation (1.62), $L_{eq} = 10\frac{\log_{10}\langle p^2 \rangle}{p_{ref}^2} = 10\log_{10}\frac{0.4675}{4 \times 10^{-10}} = 90.7$ (dB re 20 µPa).

A-weighting at 250 Hz is −8.6 dB, so $L_{Aeq} = 82.1$ (dBA re 20 µPa).

## Problem 10

(a) Taking logs of Equation (2.47), we obtain the sound level represented by the noise floor on the instrument as:
$L_p = 20\log_{10} E - S_{\mathrm{mic}} + 94 = -110 + 26 - 94 = 10$ dB (dB re 20 µPa).
If the sound level meter actually reads 13 dB, then from Example 1.41, the contribution due to the actual noise is:
$L_p = 10\log_{10}\left(10^{1.3} - 10^{1.0}\right) = 10$ (dB re 20 µPa).

(b) The ear can hear 0 dB of frontally incident sound at 2 kHz and it can probably discern signals just above its noise floor. So the equivalent sensitivity of the sound level meter would be approximately 10 dB, implying that the ear is 10 dB more sensitive.

## Problem 11

(a) A-weighted levels are calculated and tabulated below.

| | Octave band centre frequency (Hz) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 63 | 125 | 250 | 500 | 1000 | 2000 | 4000 | 8000 |
| $L_p$ (dB re 20 µPa) | 100 | 101 | 97 | 91 | 90 | 88 | 86 | 81 |
| A-weighting (dB) | -26.2 | -16.1 | -8.6 | -3.2 | 0.0 | 1.2 | 1.0 | -1.1 |
| A-weighted level (dB re 20 µPa) | 73.8 | 84.9 | 88.4 | 87.8 | 90 | 89.2 | 87 | 79.9 |

Using Equation (1.74), the overall A-weighted level is:
$L_p = 10\log_{10}\left(10^{7.38} + 10^{8.49} + 10^{8.84} + 10^{8.78} + 10^{9} + 10^{8.92} + 10^{8.7} + 10^{7.99}\right)$
$= 96.1$ (dB re 20 µPa).

(b) Octave band loudness levels in sones are listed in the following table.

| | Octave band centre frequency (Hz) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 63 | 125 | 250 | 500 | 1000 | 2000 | 4000 | 8000 |
| $L_p$ (dB re 20 µPa) | 100 | 101 | 97 | 91 | 90 | 88 | 86 | 81 |
| sones | 30 | 40 | 40 | 30 | 32 | 38 | 38 | 33 |

The overall level in sones is calculated using Equation (2.2). Thus, $L = 40 + 0.3[30 + 40 + 30 + 32 + 38 + 38 + 33] = 112$ (sones).

From Equation (2.1), $P = 40 + 10\log_2 S = 40 + \dfrac{\log_{10} S}{\log_{10} 2} = 108$ (phons).

(c) Let the contribution of the machine to the overall level be $x$ dBA. Then, using Example 1.41, $x$ is given by $x = 10\log_{10}\left(10^{96.1/10} - 10^{(96.1-1.5)/10}\right) = 90.8$ (dBA re 20 µPa).

## Problem 12

(a) Equal loudness contours show the relative response of a normal ear to the same level of incident tonal sound at each frequency (see Figures 2.1 and 2.2).

(b) Although the signal levels in the experiments to determine equal loudness contours varied substantially, the A- weighted scale corresponds approximately to the loudness contour of 60 dB. As industrial noise is usually much louder than this and equal loudness contours for high sound levels do not have the same shape as those at 60 dB, it is unlikely that the A-weighted scale will indicate correct loudness levels for most industrial noise.

## Problem 13

From Table 2.21, the adjustments to be added to the measured level prior to comparison with the base 40 dBA level are +10 dBA for nighttime operation and -15 dBA for the zoning of light industrial. Thus the corrected measured level is $L_{Aeq} = 65 - 15 + 10 = 60$ (dBA re 20 µPa), which is 20 dB above the base level of 40 dBA. From Table 2.23, the expected community response would be widespread complaints.

## Problem 14

The one third octave band levels would have to be first combined into octave band levels by logarithmically adding together three third octave bands for each octave band result. The three bands to add would be one with a centre frequency the same as the octave band and one band above and one below that one. For example, if the 200Hz, 250Hz and 315Hz one third octave band levels were 60dB, 65dB, and 63dB respectively, then, using Equation (1.74) and Table 2.4, the 250Hz octave band level would be:

$L_{p250} = 10\log_{10}\left(10^{60/10} + 10^{65/10} + 10^{63/10}\right) = 67.9$ (dB re 20 µPa).

# Problem 15

(a) See Figures 2.3 and 2.4 and read off the values as accurately as possible.

**TABLE 2.2** Results for problem 15(a)

| | Octave band centre frequency (Hz) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 63 | 125 | 250 | 500 | 1000 | 2000 | 4000 | 8000 |
| $L_p$ forward | 40 | 44 | 49 | 54 | 56 | 55 | 45 | 33 |
| $L_p$ side | 39 | 43 | 47 | 51 | 53 | 49 | 39 | 27 |
| $L_p$ rear | 38 | 42 | 44 | 48 | 49 | 44 | 27 | 15 |
| sones forward | 0 | 0.6 | 1.7 | 2.8 | 3.8 | 4.3 | 2.8 | 1.6 |
| sones side | 0.05 | 0.55 | 1.4 | 2.4 | 3.2 | 3 | 2 | 1.1 |
| sones rear | 0.02 | 0.5 | 1.1 | 2 | 2.5 | 2.2 | 0.9 | 0.3 |
| phons forward | 0 | 33 | 48 | 55 | 59 | 61 | 55 | 46.8 |
| phons side | 0 | 33 | 45 | 53 | 57 | 55.8 | 50 | 41.4 |
| phons rear | 0 | 30 | 41 | 50 | 53 | 51.4 | 39 | 22.6 |

Overall levels:
Sones Forward = 4.3 + 0.3[0.0+0.6+1.7+2.8+3.8+2.8+1.6] = 8.3
Sones Side = 3.2 + 0.3[0.05+0.55+1.4+2.4+3.0+2.0+1.1] = 6.3
Sones Rear = 2.5 + 0.3[0.02+0.5+1.1+2.0+2.2+0.9+0.3] = 4.7
Using Equation (2.1), Phons $= 40 + (10\log_{10} S)/(\log_{10} 2)$. Thus:
phons forward = 70.5
phons side = 66.6
phons rear = 62.3

(b) Yes, a person with no hearing loss would be able to hear, as the levels in the bands important for speech recognition are well above the hearing threshold level or MAF.

(c) If the distance increases to 10 m from 2 m, the sound pressure level in all bands will decrease by $20\log_{10}(10/2) = 14$ dB (assuming free field conditions). The MAF from Figure 2.1 in the textbook and the new sound pressure levels are listed in the table below.

**TABLE 2.3** results for problem 15(c)

| | Octave band centre frequency (Hz) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 63 | 125 | 250 | 500 | 1000 | 2000 | 4000 | 8000 |
| MAF | 39 | 22 | 15 | 9 | 4 | 0 | 0 | 12 |
| New $L_p$ forward | 26 | 30 | 35 | 40 | 42 | 41 | 31 | 19 |
| New $L_p$ side | 25 | 29 | 33 | 37 | 39 | 35 | 25 | 13 |
| New $L_p$ rear | 24 | 28 | 30 | 34 | 35 | 30 | 13 | 1 |

Difficulty in hearing sound in the 63 Hz and 8 kHz bands would be encountered but as these bands are not important for speech, it is expected that there would be no difficulty in understanding speech for any head orientation.

(d) In the presence of the masking noise, speech recognition is just possible for a listener looking toward the speaker, which corresponds to S = 8.3 Sones. A speaker speaking twice as loudly will increase the rear level from 4.7 to 9.4 sones, making speech recognition possible again, so the situation will be improved.

## Problem 16

**TABLE 2.4** 1/3-octave band levels used for Problem 16

| 1/3-octave centre frequency (Hz) | 1/3-octave band level (dB) | 1/3-octave centre frequency (Hz) | 1/3-octave band level (dB) | 1/3-octave centre frequency (Hz) | 1/3-octave band level (dB) |
|---|---|---|---|---|---|
| 25 | 20 | 31.5 | 35 | 40 | 45 |
| 50 | 83 | 63 | 82 | 80 | 79 |
| 100 | 77 | 125 | 81 | 160 | 78 |
| 200 | 77 | 250 | 73 | 315 | 75 |
| 400 | 71 | 500 | 68 | 630 | 64 |
| 800 | 61 | 1000 | 55 | 1250 | 71 |
| 1600 | 72 | 2000 | 74 | 2500 | 65 |
| 3150 | 61 | 4000 | 58 | 5000 | 54 |
| 6300 | 52 | 8000 | 50 | 10000 | 48 |
| 12500 | 46 | 16000 | 44 | | |

Use methods outlined in ANSI-S3.4, ISO 532-1, ISO 532-2 and ISO 532 (1975) (described above), and the associated computer programs referred to in Example 2.1.
Run the three programs that evaluate the calculation procedures in the standards, ANSI-S3.4, ISO 532-1, ISO 532-2. Also do the hand calculations described in chapter 2 of the textbook and based on ISO 532 (1975). The results are:

For the Zwicker method (ISO 532-1):

(i) Loudness level = 96.5 phons.
(ii) Loudness = 50.3 sones.

For the Moore-Glasberg method (ISO 532-2):

(i) Binaural loudness level = 96.8 phons.
(ii) Binaural loudness = 54.8 sones.

For the Moore-Glasberg method (ANSI-S3.4):

(i) Binaural loudness level = 96.9 phons.
(ii) Binaural loudness = 55.6 sones.

For the ISO 532 (1975) standard described in Chapter 2 of the textbook, we begin by reading from Figure 2.4 in the textbook, the sone level corresponding to each 1/3-octave band level in Table 2.4. The results are listed in Table 2.5.

Using Equations (2.1) and (2.2) in the textbook, together with Table 2.5, we can calculate the overall sone and phon values as well as the 1/3-octave band with the highest loudness values.

(i) Loudness level = 92.4 phons.
(ii) Loudness = 37.8 sones.
(iii) Loudest 1/3 octave band: 2000 Hz.

**TABLE 2.5** Sone levels corresponding to the 1/3-octave band levels in Table 2.4

| Frequency (Hz) | 1/3-oct. level | Sone level | Frequency (Hz) | 1/3-oct. level | Sone level | Frequency (Hz) | 1/3-oct. level | Sone level |
|---|---|---|---|---|---|---|---|---|
| 25 | 20 | 0 | 31.5 | 35 | 0 | 40 | 45 | 0.16 |
| 50 | 83 | 7.3 | 63 | 82 | 7.8 | 80 | 79 | 7.2 |
| 100 | 77 | 7 | 125 | 81 | 9.9 | 160 | 78 | 8.5 |
| 200 | 77 | 8.5 | 250 | 73 | 7.4 | 315 | 75 | 8.5 |
| 400 | 71 | 7.5 | 500 | 68 | 4.0 | 630 | 64 | 5.5 |
| 800 | 61 | 4.9 | 1000 | 55 | 3.6 | 1250 | 71 | 9.9 |
| 1600 | 72 | 11.1 | 2000 | 74 | 13.5 | 2500 | 65 | 7.8 |
| 3150 | 61 | 7 | 4000 | 58 | 5.8 | 5000 | 54 | 4.9 |
| 6300 | 52 | 4.6 | 8000 | 50 | 4.3 | 10000 | 48 | 3.6 |
| 12500 | 46 | 2.7 | 16000 | 44 | 2.0 | | | |

# 3

# Solutions to Additional Problems in Chapter 3

## Problem 1

(a) From Equation (1.64), $L_W = 10\log_{10}\dfrac{3.10}{10^{-12}} = 124.9$ (dB re $10^{-12}$ W).

(b) Upper $L_W = 10\log_{10}\dfrac{3.15}{10^{-12}} = 125.00$ (dB re $10^{-12}$ W).

Lower $L_W = 10\log_{10}\dfrac{3.05}{10^{-12}} = 124.84$ (dB re $10^{-12}$ W).

So one decimal point is too precise. The result should be given as 125 (dB re $10^{-12}$ W).

(c) From Equations (1.61) and (1.66),
$L_I = 10\log_{10}\left[\dfrac{W}{2\pi r^2}\right]+120 = 10\log_{10}\left[\dfrac{3.10}{2\pi \times 6.15^2}\right]+120 = 101$ (dB re $10^{-12}$ W/m$^2$)
where the surface is a hemisphere and why $4\pi$ in Equation (1.61) for a spherical surface is replaced with $2\pi$.

(d) From Equation (1.1), $c = \sqrt{\dfrac{\gamma RT}{M}} = \sqrt{\dfrac{1.4 \times 8.314 \times 288}{0.029}} = 340$ (m/s).

From Equation (1.1), $\rho = \dfrac{\gamma P}{c^2} = \dfrac{1.4 \times 101.4 \times 10^3}{340^2} = 1.23$ (kg/m$^3$).

Thus, $\rho c = 418$ (MKS Rayls).

(e) From Equation (1.68):
$L_p = L_I - 26 + 10\log_{10}\rho c = 101.2 - 26 + 10\log_{10}(418) = 101$ (dB re 20 µPa).

## Problem 2

(a) For each octave band, the sound pressure level in dBA is calculated using:
$L_{pA} = L_p + L_A$. The overall sound pressure level is then:
$L_{pA}(\text{overall}) = \sum_{i=1}^{7} 10\log_{10}(10^{(L_{pi}+L_{Ai})/10}$, where $L_{Ai}$ is the weighting to be added to the sound pressure level, $L_{pi}$ in the $i$th octave band to convert it to an A-weighted level. The results are shown in Table 3.1 that follows.

**TABLE 3.1** Data for Problem 2 (all dB levels are re 20 µPa)

| | Octave band centre frequency (Hz) | | | | | | | Overall (dBA) |
|---|---|---|---|---|---|---|---|---|
| | 63 | 125 | 250 | 500 | 1000 | 2000 | 4000 | |
| Tested machine on (dB) | 110.0 | 106.0 | 102.0 | 102.0 | 98.5 | 96.2 | 88.1 | |
| A-wtng (dB) | -26.2 | -16.1 | -8.6 | -3.2 | 0.0 | 1.2 | 1.0 | |
| A-weighted level (dBA) | 83.8 | 89.9 | 93.4 | 98.8 | 98.5 | 97.4 | 89.1 | 103.9 |
| Tested machine off (dB) | 109.0 | 102.0 | 97.0 | 100.0 | 95.0 | 92.1 | 86 | |
| A-weighted level (dBA) | 82.8 | 85.9 | 88.4 | 96.8 | 95.0 | 93.3 | 87 | 100.7 |
| Test M/c only (dBA) | | | | | | | | 101.0 |

(b) Allowable exposure time: $T_a = 8 \times 2^{-(103.9-85)/3} = 0.10$ (hour).

(c) Allowable exposure time: $T_a = 8 \times 2^{-(103.9-90)/5} = 1.16$ (hour).

(d) See Table 3.1 above, last line. The individual A-weighted octave band levels due to the machine alone are calculated using the procedure in Example 1.41.
$L_{pA}(\text{machine only}) = 10\log_{10}\left(10^{(\mathrm{L_p(machine\ on)-L_p(machine\ off)})/10}\right) + \mathrm{L_A}$.
The individual A-weighted octave band levels are then logarithmically summed together using Equation (1.74) to give the overall A-weighted level.

## Problem 3

(a) At 125 Hz and 20°C, wavelength, $\lambda = 343/125 = 2.74$ m, $\lambda/2 = 1.37$ m and $\lambda/6 = 0.46$ m , so measurement is in the far field as it is sufficiently far from the source in terms of wavelengths. The measurement would need to be closer than 0.46 m to be in the near field.

(b) $L_p = 91.3$ (dB re 20 µPa).

Using Equation (1.62), the sound pressure in pascals is between $10^{91.25/20} \times 2 \times 10^{-5}$ and $10^{91.35/20} \times 2 \times 10^{-5}$ Pa or between 0.730 Pa and 0.739 Pa. So use 0.735 Pa. The sound power is then obtained using Equations (1.57) and (1.61):

$$W = \frac{2\pi r^2 p^2}{\rho c} = \frac{2\pi \times 5.2^2 \times 0.735^2}{413.7} = 0.22 \text{ (W)}.$$

(c) The A-weighted sound pressure level at 4.1 m is obtained using Equations (1.57) and (1.61) and Table 2.4 as:
$91.3 + 20\log_{10}(5.2/4.1) - 16.1 = 77.3$ (dBA re 20 µPa).

## Problem 4

(a) Using equations (1.57) and (1.61) with $W$ constant, spherical spreading results in a noise reduction given by:

$\mathrm{NR} = 20\log_{10}(r_1/r_2) = 20\log_{10}(70/10) = 16.9$ dB.

(b) At 10°C, speed of sound is given by: From equation (1.1):

$$c = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.314 \times 283}{0.029}} = 337 \text{ (m/s)}.$$

From equation (1.1), Density, $\rho = \dfrac{PM}{RT} = \dfrac{101400 \times 0.029}{8.314 \times 283} = 1.25$ (kg/m$^3$).

Thus, $10\log_{10}(\rho c/400) = 10\log_{10}(337 \times 1.25) = 0.2$ dB.

As the machine is on hard ground, we may assume hemispherical radiation. We can also assume that any other effects on sound propagation such as air absorption may be ignored.
Sound power level may be calculated using Equations (1.62), (1.64) and (3.44), replacing $4\pi$ with $2\pi$ in Equation (3.44) for hemispherical radiation, taking logs and multiplying each resulting term by 10:

$$L_W = L_p + 20\log_{10} r + 8 + 10\log_{10}\frac{400}{\rho c}$$
$$= 60.0 + 36.9 + 8 - 0.2 = 104.7 \text{ (dB re } 10^{-12}\text{ W)}.$$

## Problem 5

The configuration is shown in the Figure 3.1.

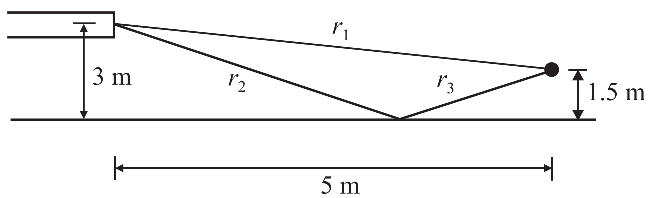


**FIGURE 3.1** Configuration for Problem 5.

(a) $r_1 = \sqrt{1.5^2 + 5^2} = 5.2201$ m; $r_2 = \sqrt{3^2 + (10/3)^2} = 4.4845$ m;
$r_3 = \sqrt{1.5^2 + (5/3)^2} = 2.2423$ (m).
$r_2 + r_3 = 6.7268$ (m).

Assume no phase change on reflection.
$r_1 - r_2 - r_3 = 1.5067$ (m).

At 40°C, speed of sound is given by Equation (1.1) as:
$$c = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.314 \times 313}{0.029}} = 354 \text{ (m/s)}.$$
Wavelength at 65 Hz $= c/f = 354/65 = 5.453$ (m).
Path length difference $= -1.5067 \times 360/5.453 = 99.47°$.

Speed of sound, from equation (1.1), $c = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{PM}{RT}}$.

Thus, $\rho = \frac{PM}{RT} = \frac{101400 \times 0.029}{8.314 \times 313} = 1.13$ (kg/m$^3$).

Direct wave pressure, from Equations (1.57) and (1.61), is:
$$p_1^2 = \frac{W\rho c}{4\pi r^2} = \frac{0.5 \times 1.13 \times 354}{4\pi \times 5.2202^2} = 0.5841 \text{ (Pa}^2\text{)},$$

and reflected wave pressure is, $p_2^2 = \frac{0.5 \times 1.13 \times 354}{4\pi \times 6.7268^2} = 0.3517$ (Pa$^2$).

From Equation (1.72) for coherent addition, $\langle p_t^2\rangle = \langle p_1^2\rangle + \langle p_2^2\rangle + 2\langle p_1 p_2\rangle \cos(\beta_1 - \beta_2)$

$= 0.5841 + 0.3517 + 2\sqrt{0.5841 \times 0.3517}\cos 99.47^\circ = 0.7866$ (Pa$^2$).

From equation (1.63), $L_p = 10\log_{10} p_t^2 + 94 = 93.0$ (dB re 20 µPa).

Assumptions:

(i) No phase shift on reflection.

(ii) No excess attenuation effects considered.

(b) Broadband sound, from Equation (1.73): $p_t^2 = p_1^2 + p_2^2 = 0.9358$ (Pa$^2$).
Thus, $L_p = 10\log_{10} p_t^2 + 94 = 93.9$ (dB re 20 µPa).

## Problem 6

(a) The total acoustic pressure, $p_t$ at location O is the sum of the two individual pressures, which are found from the velocity potential by using Equation (1.3). Thus:

$p_t = \frac{\rho A \mathrm{j}\omega}{r}\left(\mathrm{e}^{\mathrm{j}(\omega t - kr_1)} + \mathrm{e}^{\mathrm{j}(\omega t - kr_2)}\right)$

Note that from Section 3.3,
$r_1 = r - h\cos\theta$
$r_2 = r + h\cos\theta$
Thus: $p_T = \frac{A\mathrm{j}\omega\rho \mathrm{e}^{\mathrm{j}(\omega t - kr)}}{r}\left(\mathrm{e}^{\mathrm{j}kh\cos\theta} + \mathrm{e}^{-\mathrm{j}kh\cos\theta}\right)$
or

$$p_T = \frac{A\mathrm{j}\omega\rho \mathrm{e}^{\mathrm{j}(\omega t - kr)}}{r}\left[\cos(kh\cos\theta) + \mathrm{j}\sin(kh\cos\theta) + \cos(kh\cos\theta) - \mathrm{j}\sin(kh\cos\theta)\right]$$

which can be written as: $p_T = \frac{\mathrm{j}2A\omega\rho\cos(kh\cos\theta)}{r}\mathrm{e}^{\mathrm{j}(\omega t - kr)}$

(b) Using Equation (1.2) in the text, the particle velocity at location, $r$, due to one source only (as the other is far enough away to neglect) is:

$$u = -\frac{\partial\phi}{\partial r} = \frac{\mathrm{j}Ak}{r}\mathrm{e}^{\mathrm{j}(\omega t - kr)} + \frac{A}{r^2}\mathrm{e}^{\mathrm{j}(\omega t - kr)}$$

Thus the particle velocity on the surface at $r = a$ is:

$$u = \left(\frac{\mathrm{j}Ak}{a} + \frac{A}{a^2}\right)\mathrm{e}^{\mathrm{j}(\omega t - ka)} = \frac{A}{a^2}(1 + \mathrm{j}ka)\mathrm{e}^{\mathrm{j}(\omega t - ka)}$$

(c) The particle velocity amplitude is thus:

$$\hat{u} = \sqrt{\left(\frac{Ak}{a}\right)^2 + \left(\frac{A}{a^2}\right)^2} = \frac{A}{a^2}\sqrt{(ka)^2 + 1}$$

The RMS particle velocity is obtained by dividing the above expression by $\sqrt{2}$. From Example 3.4 in the textbook, the RMS source volume velocity is $4\pi \mathrm{a}^2$ multiplied by the RMS particle velocity. Thus:

$$Q = \frac{4\pi a^2}{\sqrt{2}}\frac{A}{a^2}\sqrt{(ka)^2 + 1} = \frac{4\pi}{\sqrt{2}}A\sqrt{(ka)^2 + 1}$$

For small $ka$, this simplifies to $Q = 2\pi\sqrt{2}A$.

## Problem 7

The wavenumber, $k$, is equal to $(2\pi f/c) = (2\pi \times 500/343) = 9.16$ (m$^{-1}$).
The RMS source strength, $Q$, of each monopole making up the dipole source may be calculated using Equation (3.2) in the textbook. That is, for small $ka$:

$$Q = \sqrt{\frac{4\pi W}{k^2\rho c}} = \sqrt{\frac{4\pi \times 0.01}{9.16^2 \times 1.206 \times 343}} = 1.903 \times 10^{-3} \text{ (m}^3\text{/s)}.$$

(a) The dipole intensity at $\theta = 45°$ is given by Equation (3.6) and for small $ka$, it is:

$$\begin{aligned} I_D &= \rho c \frac{k^4 h^2 Q^2}{(2\pi r)^2} \cos^2\theta \\ &= \frac{1.205 \times 343 \times 9.16^4 \times (0.005/2)^2 \times 1.903^2 \times 10^{-6}}{(2\pi \times 0.5)^2} \times 0.707^2 \\ &= 3.34\,(\mu\text{W/m}^2). \end{aligned}$$

(b) From Equation (1.57), $\langle p^2 \rangle = \rho c I = 1.205 \times 343 \times 3.33 \times 10^{-6} = 1.38 \times 10^{-3}$ (Pa$^2$).

From Equation (1.62), $L_p = 10\log_{10}\frac{\langle p^2\rangle}{p_{ref}^2} = 65.4$ (dB re 20 µPa).

## Problem 8

(a) From Equation (3.49), the average $L_p$ is:
$L_p = 10\log_{10}\{(1/5)[10^{8.5} + 10^{8.3} + 10^{8.0} + 10^{8.7} + 10^{8.6}]\} = 84.8$ (dB re 20 µPa).

(b) From Equation (3.66), the radiated power, $L_W$ is:
$L_W = L_p + 10\log_{10} S - \Delta_1 - \Delta_2 - 10\log_{10}(\rho c/400)$.
Area of measurement surface $= 2(2 \times 4 + 2 \times 3) + 3 \times 4 = 40$ (m$^2$).
Ratio of area of measurement surface to machine surface $= 40/8 = 5$, so from Table 3.8, $\Delta_2 = 0$.
Assuming that the machine is in a very large enclosure, so that there is no reverberant field to be corrected for, $\Delta_1 = 0$ .
Thus $L_W = 84.8 + 10\log_{10} 40 - 0.2 = 100.6$ (dB re $10^{-12}$ W).

## Problem 9

(a) The arrangement is shown in Figure 3.2. The speaker has been placed as closely as possible to the opening without covering any part of it.

(b) Power radiated by original opening (assuming a constant volume velocity source in a wall that is large compared to a wavelength of radiated sound) is (see Equations (3.2) and (3.4) and multiply Equation (3.2) by 2 to account for radiation into half space):

$$W_M = \frac{2Q^2_{\text{RMS},H}\rho c k^2}{4\pi(1 + k^2a^2)}$$

The power radiated by the dipole may be calculated using Equation (3.7). Again this equation must be multiplied by 2. Thus:

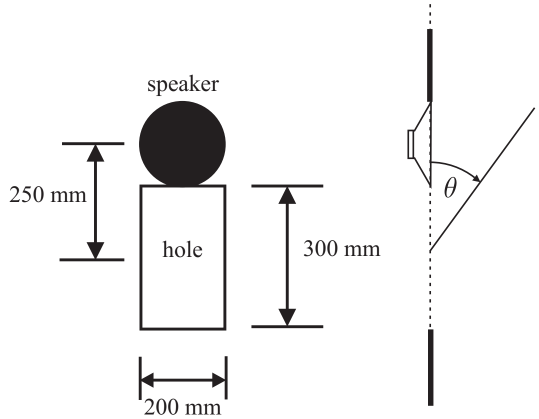


**FIGURE 3.2** Arrangement for Problem 9.

$$W_D = \frac{2\rho c h^2 Q^2_{\mathrm{RMS},H} k^4}{3\pi(1+k^2a^2)}$$

The wavenumber, $k = 2\pi \times 125/343 = 2.290$ ($\mathrm{m}^{-1}$).

The ratio, $\dfrac{W_D}{W_M} = \dfrac{4}{3}h^2k^2 = \dfrac{4}{3} \times 0.25^2 \times 2.29^2 = 3.6$ (dB).

(c) The ratio of the mean square pressures may be obtained using Equations (3.3) and (3.18). Thus:

$$\frac{\langle p_D^2 \rangle}{\langle p_M^2 \rangle} = \frac{[3\rho c W_D \cos^2\theta / 4\pi r^2] \times 2}{[W_M \rho c/4\pi r^2] \times 2} = 3\cos^2\theta \frac{W_D}{W_M} = 4h^2k^2\cos^2\theta = 0.57\cos^2\theta$$

The reduction in decibels is $-10\log_{10}[0.57\cos^2\theta]$.

Values of sound pressure level reduction (dB re 20 $\mu$Pa) are tabulated in Table 3.2 for the required values of $\theta$.

**TABLE 3.2** Table for Problem 9

| | $\theta = 0$ | $\theta = \pi/4$ radians | $\theta = \pi/2$ radians |
|---|---|---|---|
| dB Reduction in SPL | 2.4 | 5.5 | $\infty$ |

When the speakers are turned on, the sound field amplitude distribution has 2 lobes with maxima at $\theta = 0$, $\pi$ and one minimum at $\theta = \pi/2$, which is a typical dipole radiation pattern (for radiation into half space). The radiation is illustrated in Figure 3.3.

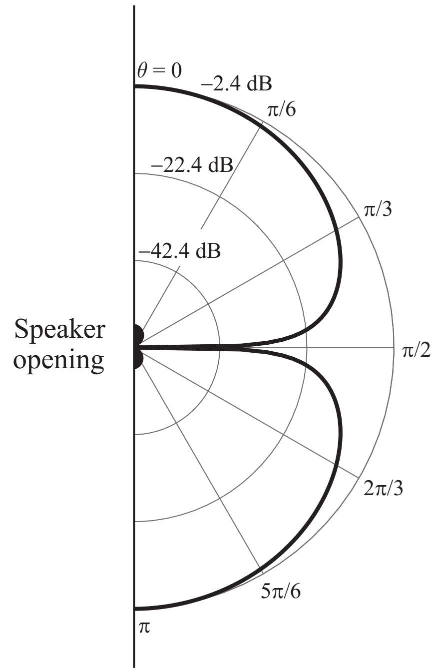


**FIGURE 3.3** Radiation Pattern for Problem 9.

## Problem 10

The traffic may be treated as an incoherent, infinite line source as the traffic noise sources are completely independent. For a source separation distance, $b$ of 8 m, and a sound pressure level measurement at a distance, $r_1 = 1$ m, from a single source, (such that $r_1 < b/2$), the mean square sound pressure is related to the sound power, $W$, of a single vehicle by Equation (3.22). Thus:

$$\langle p_1^2 \rangle = \rho c \frac{W}{4\pi r_1^2} \times \text{DF}$$

where DF is the directivity factor for the source/ground combination. At $r_2 = 200$ m, the sound pressure is related to the sound power, $W$, of one vehicle by Equation (3.23). Thus:

$$\langle p_2^2 \rangle = \rho c \frac{W}{4 b r_2} \times \text{DF}$$

Thus:

$$\frac{\langle p_1^2 \rangle}{\langle p_2^2 \rangle} = \frac{b r_2}{\pi \times r_1^2} = \frac{6 \times 200}{\pi} = 382$$

The level at 200 m = level at 1 m $-10\log_{10} \dfrac{\langle p_1^2 \rangle}{\langle p_2^2 \rangle} = 86 - 25.8 = 60.2$ (dBA re 20 µPa).

## Problem 11

(a) Piston radiating a sound power of 100 W at 200 Hz from a baffle which may be assumed to act as a baffle of infinite size as it is larger than or equal to 3 wavelengths. $ka = 2\pi \times fa/c = 2\pi \times 200 \times 0.2/343 = 0.733$ and $a = 0.2$ m.

From Equation (3.35) or Figure 3.16, for $ka = 0.733$, $R_R(2ka) = 0.246$.
The radiated sound power in terms of the piston velocity amplitude is given by Equation (3.33) as:

$$W = R_R\,\pi a^2 \rho c \hat{u}^2/2$$

Thus, the piston velocity amplitude, $U = \hat{u}$, is:

$$\hat{u} = \sqrt{2W/(R_R\,\pi a^2 \rho c)} = \sqrt{2\,\times\,100/(0.246 \times \pi \times 0.2^2 \times 413.6)} = 3.96 \text{ m/s}.$$

(b) From equation (3.31), the on-axis intensity (in direction $\theta = 0$) is:

$$I = \frac{\rho c k^2}{8\pi^2 r^2}\,F^2(q)$$

where $k = 2\pi \times 200/343 = 3.66$, $r = 5$, $a = 0.2$ and $q = ka\sin\theta = 0$.
From Figure 3.14, $F(q = 0) = U\pi a^2$.
Thus:

$$I = \frac{\rho c k^2 U^2 \pi^2 a^4}{8\pi^2 r^2} = \frac{\rho c k^2 U^2 a^2}{8r^2}$$

$$= \frac{1.206\,\times\,343\,\times\,3.66^2\,\times 3.96^2\,\times\,0.04}{8\,\times\,5^2} = 17.4\,\text{W/m}^2.$$

(c) Radiation mass loading $= \pi a^2 \rho c[X(2ka)]$, where $X(2ka) = 0.54$ (see Figure 3.16). Thus the mass loading $= \pi\,\times\,0.04\,\times\,1.206\,\times\,343\,\times\,0.54 = 28.1$ kg/s.

(d) Sound pressure level at 5 m:

$$L_p = 10\log_{10}\frac{\langle p^2\rangle}{p_{ref}^2} = 10\log_{10}\frac{\rho c I}{4\,\times\,10^{-10}} = 10\log_{10}\frac{413.6 \times 17.4}{4\,\times\,10^{-10}} = 132.6 \text{ (dB re 20 µPa)}$$

## Problem 12

(a) The arrangement is illustrated in Figure 3.4.

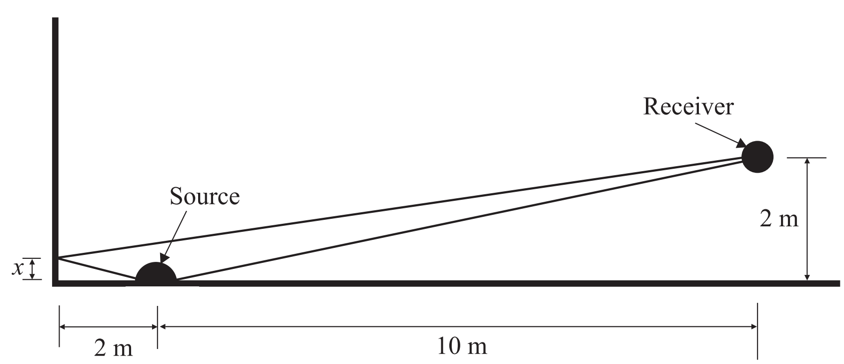


**FIGURE 3.4** Arrangement for Problem 12.

The sound pressure level at the receiver is obtained by logarithmic addition of the sound level due to the ray arriving directly from the source and the sound level due to the ray reflected by the wall. As the source is mounted on a hard floor, we may assume hemispherical radiation so that for the direct ray, taking logs of Equation (3.48) with $D = 2$ and multiplying both sides by 10 gives:

$$L_{p,\text{direct}} = L_W + 10\log_{10}(2\pi \times 10^2) + 10\log_{10}(\rho c/400)$$

$$= 115 - 10\log_{10}(200\pi) + 10\log_{10}(413/400) = 87.2 \text{ (dB re } 20\,\mu\text{Pa)}.$$

The distance travelled by the ray reflected from the wall is $r = \sqrt{(2+x)^2 + 12^2}$. The value of $x$ is found using similar triangles so that $\frac{x}{2} = \frac{2}{10}$, so $x = 0.4$ m.

Thus $r = 12.24$ m. The corresponding sound pressure level is then:

$$L_{p,\text{reflected}} = L_W + 10\log_{10}(2\pi \times 12.24^2) + 10\log_{10}(\rho c/400)$$
$$= 115 - 10\log_{10}(299.5\pi) + 10\log_{10}(413/400) = 85.4 \text{ (dB re } 20\,\mu\text{Pa)}.$$

Using Equation (1.74), the total sound pressure level at the receiver is:

$$L_p = 10\log_{10}(10^{87.2/10} + 10^{85.4/10}) = 89.4 \text{ (dB re } 20\,\mu\text{Pa)}.$$

(b) Possible reasons for the sound level being 2 dB higher than calculated are:

(i) The source is not a constant power source and the floor and wall could increase its power output if it were a constant volume velocity source or had some properties of such a source.

(ii) There could be other reflecting surfaces that were not taken into account in the calculations.

(iii) The assumption of uniform radiation may not be correct.

## Problem 13

The opening may be treated as a plane incoherent sound source. The sound power level radiated by the opening can be calculated from the sound pressure level (90 dB) at 1 m from the opening using Equation (3.38) or Figure 3.18. If Figure 3.18 is used, the sound power level is first calculated for a hemispherical point source (as the opening is in a large plane surface) and the figure is then used to add decibels to that result to obtain the sound power level for the plane source. Then the sound pressure level at 200 m is calculated assuming a point hemispherical source and there is no correction needed at this distance (as the plane source will appear as a point source at large distances from it). The normalised distance to use in the figure for the 1 m location is $1/\sqrt{2} = 0.71$. In the figure, $\alpha = 2$. Thus the correction needed is $-2$ dB, which is the reduction in sound pressure level of the plane source (compared to a monopole when both sources have same sound power level).

Assuming a hemispherical point source, the sound power level is given by Equation (3.3) with the term, $4\pi r^2$ replaced with $2\pi r^2$ to account for hemispherical radiation as:

$$L_W = L_p + 10\log_{10}(2\pi r^2) - 0.14 + 2 = 90 + 8 - 0.1 + 2 = 99.9 \text{ (dB re } 10^{-12}\,\text{W)}.$$

Using Equation (3.38)

$$L_W = 10\log_{10}\left[\frac{\langle p^2\rangle \pi H L}{2\rho c \arctan\left(\dfrac{HL}{2r\sqrt{H^2+L^2+4r^2}}\right)}\right] + 120 \text{ (dB re } 10^{-12}\,\text{W)}.$$

$\langle p^2\rangle = 400 \times 10^{-12} \times 10^9 = 0.4$ Pa$^2$, $H = 1$ m, $L = 2$ m, $r = 1$ m, $\rho c = 413.6$ kg m$^{-2}$ s$^{-1}$. Thus:

$$L_W = 10\log_{10}\left[\frac{0.4 \times \pi \times 2}{2 \times 413.6 \times \arctan\left(\dfrac{2}{2 \times 1 \times \sqrt{1^2+2^2+4}}\right)}\right] + 120 = 99.8 \text{ (dB re } 10^{-12}\,\text{W)}.$$

The sound pressure level at 200 m may be calculated assuming that the plane source acts as a hemispherical point source with a sound power level of 99.8 (dB re $10^{-12}$ W). Thus the sound pressure level at 200 m is:

$L_p = 99.8 - 10\log_{10}(2\pi \times 400^2) + 0.14 = 39.9$ (dB re 20 µPa).

## Problem 14

For a constant power source, mounting the source on a hard floor will not affect its sound power output. Assuming that the reverberant field dominates the direct field over most of the room, we can use Equation (3.54) to relate the space average sound pressure level, $L_p$ to the source sound power level, $L_W$. Thus:

$$L_p = L_W - 10\log_{10} V + 10\log_{10} T_{60} - 10\log_{10}(1 + S\lambda/8V) + 13.9 \qquad \text{(dB re 20 µPa)}.$$

Room volume, $V = 3 \times 4 \times 5 = 60$ (m$^3$).
Room surface area, $S = 2(3 \times 4 + 4 \times 5 + 3 \times 5) = 94$ (m$^2$).
Wavelength, $\lambda = 343/500 = 0.686$ (m).
Thus, the space average $L_p$ is:

$$L_p = 125 - 10\log_{10}[60] + 10\log_{10}[4.5] - 10\log_{10}[1 + 94 \times 0.686/(8 \times 60)] + 13.9$$
$$= 127.1 \text{ (dB re 20 µPa)}.$$

## Problem 15

We can use Equation (3.65) for this calculation. The area of the smaller test surface (excluding the floor) is:

$S_1 = 2[(2+4) \times (1+2) + (1+4) \times (1+2)] + (2+4) \times (1+4) = 90$ (m$^2$).

$S_2 = 2[(2+8) \times (1+4) + (1+8) \times (1+4)] + (2+8) \times (1+8) = 280$ (m$^2$).

$$L_W = 80 - 10\log_{10}[(1/90) - (1/280)] + 10\log_{10}\left[10^{(84-82)/10} - 1\right] - 0.14$$
$$= 98.8 \text{ (dB re } 10^{-12} \text{ W)}.$$

# 4

# Solutions to Additional Problems in Chapter 4

## Problem 1

(a) Referring to the analysis In Section 4.5.2 in the textbook, for normal incidence, $\theta = 0$. Thus, from Equation (4.12), $\psi = 0$ and Equation (4.11) becomes (with $Z_m = \rho_2 c_2 = 830$):

$$R_p = \frac{830 - \rho c}{830 + \rho c} = \frac{830 - 413.6}{830 + 413.6} = 0.335$$

From Equation (1.62), the RMS sound pressure of the incident wave is:

$$p_i = 2 \times 10^{-5} \times 10^{60/20} = 0.02 \text{ (Pa)}$$

Thus the RMS sound pressure of the reflected wave is:

$$p_r = 0.335 p_i = 0.0067 \text{ (Pa)}$$

(b) When all the energy is reflected, $|R_p| = 1$. Thus:

$$\rho_2 c_2 \cos\theta - \rho c \cos\psi = \rho_2 c_2 \cos\theta + \rho c \cos\psi$$

Using Snell's Law (Equation (4.13) in the textbook):

$$\sin\psi = \frac{c_2 \sin\theta}{c}$$

Thus:

$$\cos\psi = \left(1 - \frac{c_2^2}{c^2}\sin^2\theta\right)^{1/2}$$

Substituting this result into the previous equation gives:

$$\rho_2 c_2 \cos\theta - \rho c \left(1 - \frac{c_2^2}{c^2}\sin^2\theta\right)^{1/2}$$

$$= \rho_2 c_2 \cos\theta + \rho c \left(1 - \frac{c_2^2}{c^2}\sin^2\theta\right)^{1/2}$$

This is true only if $(c_2/c)\sin\theta = 1$, or if $\theta = \sin^{-1}(c/c_2)$, which is the angle above which all incident energy will be reflected.

## Problem 2

(a) Sound power level of the opening is given by the hemispherical radiation version of Equation (3.4), where $S = 2\pi r^2$:

$L_W = L_p + 10\log_{10}(S) - 0.15 = 100 + 9.54 - 0.15 = 109.4$ (dB re $10^{-12}$ W).

Looking at Figure 3.18 in the textbook, it is clear that we can treat it as a point source.

Thus the sound pressure level is given by Equations (4.2) and (4.3), with $r = d_{\mathrm{SR}}$, so that:

$L_p = L_W - 10\log_{10}(2\pi r^2) - A_E = 109.4 - 62 + 0.15 - A_E = 47.6 - A_E$ (dB re 20 µPa).

The quantity, $A_E$, is made up of air absorption, ground effect and meteorological effects. The weather variability makes the air absorption vary from 1.2 to 1.5 dB so the variation can be attributed chiefly to meteorological effects. The ground effect is calculated using Figure 4.3 in the textbook and Figure 4.1 below to calculate $\beta$.

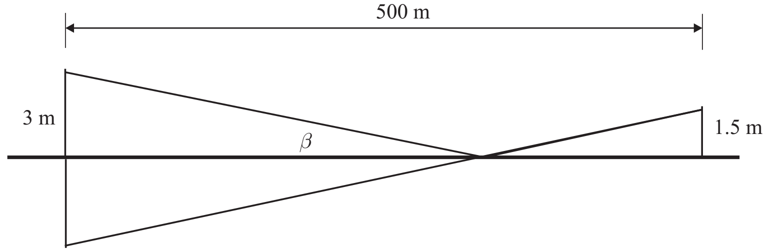


**FIGURE 4.1** Arrangement for Problem 2.

From Figure 4.1, $\beta = \sin^{-1}(3/(0.667 \times 500)) = 0.516°$.

From Table 4.8 in the textbook, $R_1 = 2 \times 10^5$, so:

$\beta[R_1/\rho f]^{1/2} = 0.516 \times [200,000/(1.205 \times 500)]^{1/2} = 9.4$ , and

$\rho f/R_1 = 1.205 \times 500/200000 = 0.003$.

From Figure 4.4 in the textbook, $A_{rf} = 1$ dB and the ground effect is:

$A_{\mathrm{gr}} = -10\log_{10}\left[1 + 10^{-1/10}\right] = -2.5$ (dB).

From Table 4.3 in the textbook, it can be seen that meteorological effects vary $+7, -5$, so the sound level with zero meteorological influence would be 51 dB. Including air absorption and ground effects, the opening is contributing 47.6 - 1.3 + 2.5 = 48.8 (dB re 20 µPa).

(b) Without the opening, we would expect the sound level to be:

$10\log_{10}\left(10^{51/10} - 10^{48.8/10}\right) = 47$ (dB re 20 µPa).

So we would expect a 4 dB noise reduction.

## Problem 3

(a) From Equation (4.14) and as given in the question, $R_p = \dfrac{Z_s - \rho c}{Z_s + \rho c}$

From Equation (5.31), $\bar{\alpha} = 1 - |R_p^2| = 1 - \dfrac{|Z_s - \rho c|^2}{|Z_s + \rho c|^2} = \dfrac{|Z_s + \rho c|^2 - |Z_s - \rho c|^2}{|Z_s + \rho c|^2}$

Rearranging gives:

$$\bar{\alpha} = \frac{[\mathrm{Re}\{Z_s\} + \rho c]^2 + [\mathrm{Im}\{Z_s\}]^2 - [\mathrm{Re}\{Z_s\} - \rho c]^2 - [\mathrm{Im}\{Z_s\}]^2}{|Z_s + \rho c|^2}$$

$$= \frac{[\mathrm{Re}\{Z_s\}]^2 + 2\rho c\mathrm{Re}\{Z_s\} + (\rho c)^2 - [\mathrm{Re}\{Z_s\}]^2 + 2\rho c\mathrm{Re}\{Z_s\} - (\rho c)^2}{|Z_s + \rho c|^2}$$

$$= \frac{4\rho c\,\mathrm{Re}\{Z_s\}}{|Z_s + \rho c|^2}$$

(b) From the above expression it can be seen that the maximum value of $\bar{\alpha}$ is 1 which would occur when $Z_s = \rho c$.

## Problem 4

Using Equation (1.65), the power radiated by the window = 0.1 W = 110 dB re $10^{-12}$ W.
Concrete ground, so $A_{\mathrm{gr}} = -3$ dB.
Range due to meteorological conditions is: (+8, -6 dB) (Table 4.3 in the textbook).

Assume:

1. Temperature is 20°C, so $A_{\mathrm{atm}}$ ranges from 2.6 to 2.8. Use $A_{\mathrm{atm}} = 2.7$.
2. No barriers exist between the source and receiver.
3. The receiver is far enough away for the window to be treated as a point source in a baffle. Thus, using Equations (4.1) and (4.3), and Table 4.3:

$$L_p = L_W - 10\log_{10}(2\pi r^2) - A_{\mathrm{gr}} - A_{\mathrm{atm}} - A_{\mathrm{met}} = 110 - 65.5 + 3 - 2.7 \times 0.75 - A_{\mathrm{met}}$$
$$= 45.5\ (+8, -6)\ (\text{dB re } 20\,\mu\text{Pa}).$$

## Problem 5

(a) From Equation (3.20), for an infinite line source at an ambient temperature of 20°C

$$\langle p^2 \rangle = \rho c \frac{W}{4br_0} = \frac{413.6 \times 0.1}{4 \times 10 \times 100} = 0.01034\ (\mathrm{Pa}^2).$$

From Equation (1.63), the sound pressure level is:

$L_p = 10\log_{10}\langle p^2 \rangle + 94 = 74.1$ (dB re 20 µPa).

(b) From Figure 4.2 below, $\beta = \sin^{-1}(1.5/75) = 1.146°$.

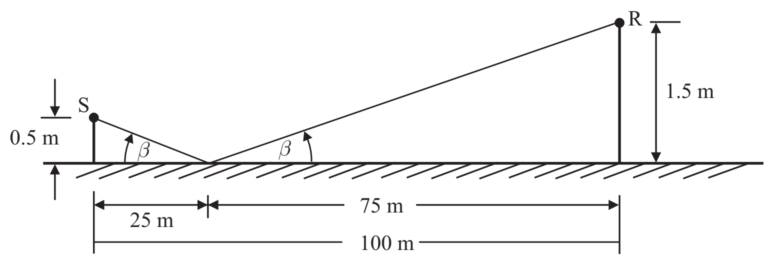


**FIGURE 4.2** Arrangement for Problem 5(b).

Input data given in the problem.
$R_1 = 2 \times 10^5$.
$\rho = 1.206$.
$f = 500$ Hz.

$$\frac{\rho f}{R_1} = 3 \times 10^{-3}; \quad \beta \left(\frac{R_1}{\rho f}\right)^{1/2} = 20.9.$$

From Figure 4.3 in the textbook, reflection loss, $A_{rf} = 1.5$ dB.

Thus, from Equation (4.15), the ground effect is:
$A_{\text{gr}} = -10 \log_{10} \left[1 + 10^{-1.5/10}\right] = -2.3$ (dB).

From Table 4.1, the attenuation due to atmospheric absorption is 2.4 dB per 1000 m. The source receiver distance is 100 m so the attenuation due to atmospheric absorption is 0.2 dB.

Thus the resulting sound pressure level at the community location is:
$L_p = 74.1 + 2.3 - 0.2 = 76.2$ dB.

(c) Strength of wind for community to be in shadow zone.
Use Equation (4.25) in the textbook, where $h_S = 0.5$ and $h_R = 1.5$. Thus:
$x = \sqrt{-2R_c}\left[\sqrt{h_S} + \sqrt{h_R}\right] = 100$ m
which gives $\sqrt{-2R_c} = 51.76$ or $R_c = -1339.7$ m.

Using Equation (4.16) in the textbook and Figure 4.3,

$$\left[\frac{\mathrm{d}c}{\mathrm{d}h}\right] = -\frac{343}{1339.7 \times (1339.7 - 0.5)/1339.7} = -0.256 \text{ s}^{-1}$$

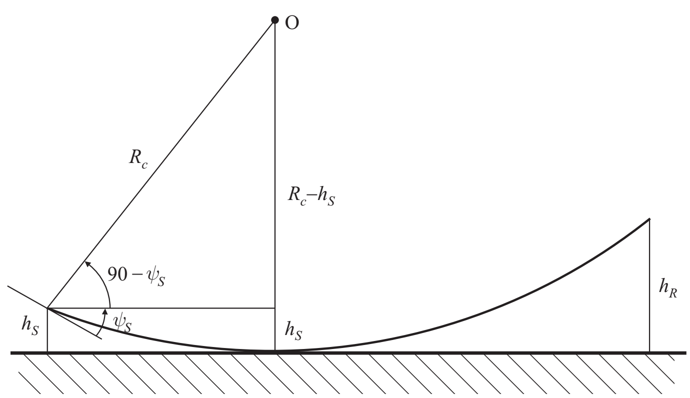


**FIGURE 4.3** Arrangement for Problem 5(d).

Using Equation (4.22) in the textbook, $\left[\frac{\mathrm{d}U}{\mathrm{d}h}\right] = \left[\frac{\mathrm{d}c}{\mathrm{d}h}\right] = \xi \frac{U}{10} = 0.15 \frac{U}{10}$

Thus U = 17 m/s which is the wind speed from receiver to source at a height of 10 m, which will put the receiver in the shadow zone.

## Problem 6

(a) ISO9613 calculations.
As the process equipment is 500 m from the receiver, it may be treated as a point source. We may use Equations (4.33) and (4.34) in the textbook, with $\text{DI}_{ik} = 0$. The

sound power levels are different for each frequency band, as are some of the excess attenuations of Equation (4.34) in the textbook.

$A_{\text{div}} = 10\log_{10}(4\pi) + 10\log_{10}(500) = 65.0$ dB for all frequencies.

From Table 4.1 in the textbook,
$A_{\text{atm}}(500) = 1.3$ dB; $A_{\text{atm}}(1000) = 2.7$ dB; $A_{\text{atm}}(2000) = 4.7$ dB.

The ground effect excess attenuation, $A_{\text{gr}}$, is calculated as described in Section 4.8.1 in the textbook.
For concrete ground, $G_g = 0$ for all frequencies.
For normal uncompacted ground, $G_g = 1.0$, so $G_R = G_m = 1.0$ for all frequencies.
Concrete extends 50 m from the source, so $G_S = [(30 \times 4) - 50]/(30 \times 4) = 0.583$ for all frequencies.
Thus, $A_S(500) = -1.5 + 0.583 \times [1.5 + 14.0 \times \mathrm{e}^{-0.46\times 4^2} \times (1 - \mathrm{e}^{-500/50})] = -0.62$ dB.
$A_S(1000) = -1.5 + 0.583 \times [1.5 + 5.0 \times \mathrm{e}^{-0.9\times 4^2} \times (1 - \mathrm{e}^{-500/50})] = -0.63$ dB.
$A_S(2000) = -1.5 \times (1 - 0.583) = -0.6$ dB.
$A_{\text{Rec}}(500) = -1.5 + [1.5 + 14.0 \times \mathrm{e}^{-0.46\times 4^2} \times (1 - \mathrm{e}^{-500/50})] = 0$ dB.
$A_{\text{Rec}}(1000) = -1.5 + [1.5 + 5.0 \times \mathrm{e}^{-0.9\times 4^2} \times (1 - \mathrm{e}^{-500/50})] = 0$ dB.
$A_{\text{Rec}}(2000) = -1.5 \times (1 - 1) = 0$ dB.
$A_{\text{mid}} = 0$ for all frequencies.
Thus $A_{\text{gr}}(500) = A_S(500) = -0.6$ dB.
$A_{\text{gr}}(1000) = A_S(1000) = -0.6$ dB.
$A_{\text{gr}}(2000) = A_S(2000) = -0.6$ dB.
Meteorological effects are already included for octave band calculations so no additional amount is added (as this is not a long time averaged A-weighted calculation).

There are no other excess attenuation terms to be included. Thus the octave band sound pressure levels at the receiver due to the proposed process equipment as well as the total expected A-weighted noise level due to the existing plus proposed equipment are listed in Table 4.1 below. The new equipment contribution is added to the new process equipment contribution using Equation (1.74) in the textbook.

**TABLE 4.1** Results for Problem 7(a)

| | Octave band centre frequency (Hz) | | | Overall |
|---|---|---|---|---|
| | 500 | 1000 | 2000 | |
| $L_W$ (dB re $10^{-12}$ W) | 121 | 119 | 117 | |
| $A_{\text{div}}$ (dB) | 65 | 65 | 65 | |
| $A_{\text{atm}}$ (dB) | 1.3 | 2.7 | 4.7 | |
| $A_{\text{gr}}$ (dB) | −0.6 | −0.6 | −0.6 | |
| New equipment $L_p$ (dB re 20 µPa) | 55.3 | 51.9 | 47.9 | |
| New equipment $L_p$ A-weighted (dB re 20 µPa) | 52.1 | 51.9 | 49.1 | 56.0 |
| Total $L_p$ (new+old) A-weighted (dB re 20 µPa) | | | | 56.2 |

(b) CONCAWE calculations
The values for $A_{\text{div}} = K_1$ and $A_{\text{atm}} = K_2$ are the same as for the ISO9613 method. As the hard ground covers one tenth of the total distance, between source and receiver, it makes sense to use the soft ground model. Thus, $K_3 = 0$ dB.

From Tables 4.6 and 4.7 in the textbook, the meteorological category is 6.
From Figure 4.11 in the text, the excess attenuation due to meteorological effects in the 500 Hz, 1000 Hz and 2000 Hz octave bands is −5.5, −5 and −4.5 dB, respectively. All other excess attenuation effects are zero. The results are summarised in Table 4.2.

**TABLE 4.2** Results for Problem 7(b)

| | Octave band centre frequency (Hz) 500 | 1000 | 2000 | Overall A-weighted |
|---|---|---|---|---|
| $L_W$ (dB re $10^{-12}$ W) | 121 | 119 | 117 | |
| $A_{\text{div}}$ (dB) | 65 | 65 | 65 | |
| $A_{\text{atm}}$ (dB) | 1.3 | 2.7 | 4.7 | |
| $A_{\text{gr}}$ (dB) | 0 | 0 | 0 | |
| $A_{\text{met}}$ (dB) | −5.5 | −5.0 | −4.5 | |
| New equipment $L_p$ (dB re 20 µPa) | 60.2 | 56.3 | 51.8 | |
| New equipment $L_p$ A-weighted (dB re 20 µPa) | 57.0 | 56.3 | 53.0 | 60.5 |
| Total $L_p$ (new+old) A-weighted (dB re 20 µPa) | | | | 60.6 |

(c) According to ISO 9613, a reduction of 11.2 dBA would be required.
According to CONCAWE, a reduction of 15.6 dBA would be required.

(d) Effect of foliage of width 50 m with one edge 10 m from receiver.
First determine the path length of the ray travelling through foliage. The receiver height is 1.5 m and the assumed radius of curvature of the ray path is 5000 m. The arrangement is shown in Figure 4.4.

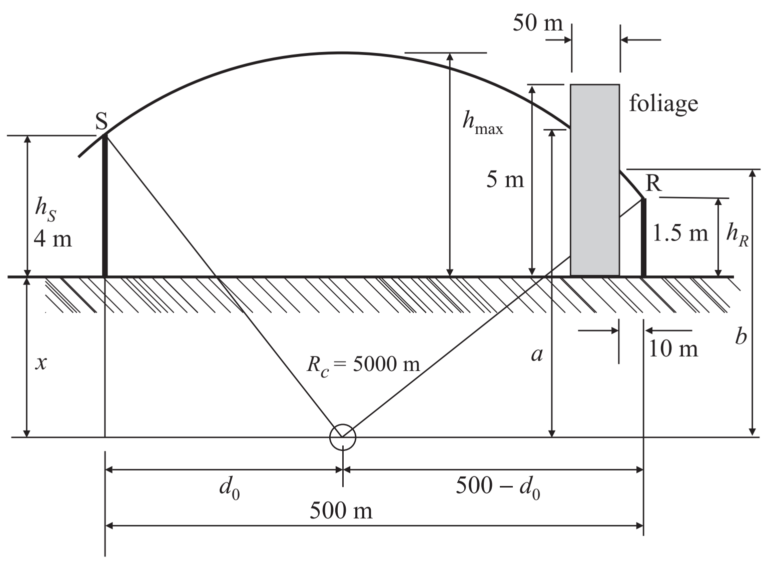


**FIGURE 4.4** Arrangement for Problem 7(d).

Inspection of Figure 4.4 leads to the following equations.
$(4 + x) = \sqrt{5000^2 - d_0^2}$
$(1.5 + x) = \sqrt{5000^2 - (500 - d_0)^2}$
Subtracting the second equation from the first gives:
$2.5 = \sqrt{5000^2 - d_0^2} - \sqrt{5000^2 - (500 - d_0)^2}$
Solving by trial and error gives $d_0 = 225.03$ m.
Substituting for $d_0$ in the first of the above equations gives $x = 4990.9$ m.

In Figure 4.4, $a = \sqrt{5000^2 - (500 - 60 - 225.03)^2} = 4995.4$ and $(a - x) = 4.5$ m. Thus the ray travels the full 50 m width of the foliage. From Table 4.10 in the textbook, $A_{\text{fol}}(500) = 2.5$ dB; $A_{\text{fol}}(1000) = 3$ dB; $A_{\text{fol}}(2000) = 4$ dB.

Using Equation (1.74) (adding the A-weighting and subtracting the foliage attenuation), the new A-weighted noise level for new process equipment would be:
$10\log_{10}\left(10^{(55.3-2.5-3.2)/10} + 10^{(51.9-3.0)/10} + 10^{(47.9+1.2-4.0)/10}\right)$
$= 53.0$ (dBA re 20 µPa).
So the noise reduction for the new equipment is $56.0 - 53.0 = 3.0$ dBA. Assume that the existing equipment has the same A-weighted noise reduction so, from Equation (1.74), the new total noise level would be $10\log_{10}\left(10^{53.0/10} + 10^{(42-3.0)/10}\right) = 53.2$ (dBA re 20 µPa), which is a reduction of 3.0 dBA.

(e) The required reduction of a wall is $53.0-45 = 8.0$ dBA. Use 8.5 dBA to be conservative and assume that the reduction of 8.5 dB at 500 Hz will result in an overall 8.5 dBA or more as frequencies higher than 500 Hz will be attenuated more and levels at lower frequencies are much lower than the 500 Hz level and thus will have a negligible effect on the A-weighted level.
We may use Equation (4.43) in the textbook.
In this equation, $C_2 = 20$, $\lambda = 343/500 = 0.686$ m, $C_3 = 0$.
$K_{\text{met}}$ and $\Delta z_1$ may be calculated using the dimensions in Figure 4.5 below.

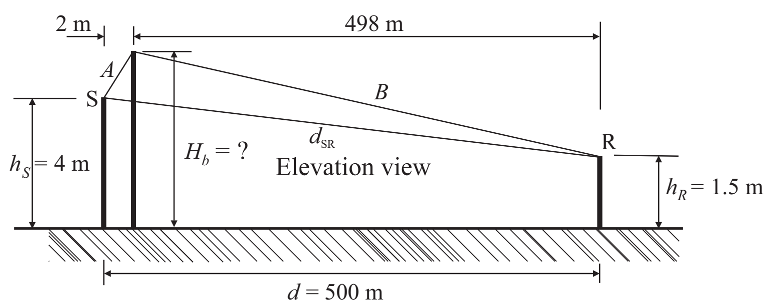


**FIGURE 4.5** Arrangement for Problem 7(e).

Begin with a barrier height of 6 m. Wavelength at 500 Hz = 343/500 = 0.686 m.
From Figure 4.5, $d_{\text{SR}} = \sqrt{500^2 + (4 - 1.5)^2} = 500.006$ m,
$A = \sqrt{(6 - 4)^2 + 2^2} = 2.828$ m, and
$B = \sqrt{498^2 + (6 - 1.5)^2} = 498.020$ m.
From Equation (4.51):
$$K_{\text{met}} = \exp\left[-\frac{1}{2000}\sqrt{\frac{2.828 \times 498.02 \times 500.006}{2(2.828 + 498.02 + 0 - 500.006)}}\right] = 0.724 \quad \text{and}$$
$\Delta z_1 = A + B - d_{\text{SR}} = 2.828 + 498.02 + 0 - 500.006 = 0.843$ m.
From Equation (4.43) in the textbook, the excess attenuation due to the barrier at 500 Hz is:

$D_{z1} = 10\log_{10}[3 + (20/0.686) \times 0.724 \times 0.843] = 13.2$ dB, which is more than required. So we will try $H_b = 5$ m. In this case, $D_{z1} = 8.5$ dB which is acceptable.
The required wall height to satisfy the allowed noise level calculated using the ISO 9613 model is 5.0 m.

## Problem 7

(a) From Equation (1.74), the total A-weighted sound pressure level is:
$L_{pA} = 10\log_{10}\left[10^{38/10} + 10^{40/10} + 10^{42/10}\right] = 45.1$ (dBA re 20 µPa).

(b) Assuming that the standard uncertainty follows a normal distribution, it will be equal to (3/2) dB for the noise level predictions made using ISO 9613, for each source. The standard uncertainty for the sound power level measurements are (2/2) dB. The uncertainty associated with the sound pressure level prediction for source, $i$, is given by Equation (4.58) in the textbook. Thus:

$u_{s,i} = \sqrt{1^2 + 1.5^2} = 1.80$ (dB).

The overall standard uncertainty for the total predicted sound pressure level is given by Equation (4.57) in the textbook. Thus, Setting the 38 dBA source as producing $L_{p1}$, we have:

$$u_{s,\mathrm{tot}} = \frac{\sqrt{(1.8 \times 10^{38/10})^2 + (1.8 \times 10^{40/10})^2 + (1.8 \times 10^{42/10})^2}}{10^{38/10} + 10^{40/10} + 10^{42/10}} = 1.1 \text{ (dB)}.$$

To obtain the 95% confidence limits, we multiply the standard uncertainty by a factor of 2 for a normal distribution. Thus the 95% confidence limits are ±2.2 dB.

If we assumed a rectangular distribution, the standard uncertainties for the sound pressure level predictions and sound power level measurements would be 1.73 and 1.15 dB, respectively. Thus $u_{s,i} = 2.08$ dB, $u_{s,\mathrm{tot}} = 1.3$ dB and the 95% confidence limits would be ±2.6 dB, indicating that the assumption of the distribution type is not very important in this case.

## Problem 8

(a) The wall arrangement is shown in Figure 4.6.
Let $i = 1$ for the diffraction path over the top of the barrier and $i = 2,\ 3$ for diffraction around the two ends. The quantities in Equation (4.43) in the textbook are:

$C_2(\text{top}) = 20$, $C_2(\text{sides}) = 40$, $C_3 = 0$ (single wall).

$A_1 = \sqrt{2.5^2 + 1^2} = 2.693$ m, $A_2 = A_3 = \sqrt{2.5^2 + 3^2} = 3.905$ m.

$B_1 = \sqrt{1.5^2 + 37.5^2} = 37.530$ m, $B_2 = B_3 = \sqrt{3^2 + 37.5^2} = 37.620$ m.

$d_{\mathrm{SR}} = \sqrt{40^2 + (2 - 1.5)^2} \approx d = 40.0$ m.

Thus, $\Delta z_1 = A_1 + B_1 - d_{\mathrm{SR}} = 0.223$, $\Delta z_2 = \Delta z_3 = A_2 + B_2 - d_{\mathrm{SR}} = 1.525$ m.

From Equation (4.51), $K_{\mathrm{met}}(\text{top}) = \exp\left[-\dfrac{1}{2000}\sqrt{\dfrac{2.693 \times 37.530 \times 40.0}{2(2.693 + 37.53 - 40.0)}}\right] = 0.953.$

$K_{\mathrm{met}}(\text{sides}) = 1.0$ (not calculated for side diffraction).

The results calculated using Equation (4.43) in the textbook are provided as a function of frequency in Table 4.3. The overall noise reduction is calculated for each frequency band by combining the reductions logarithmically. As an example, the overall noise

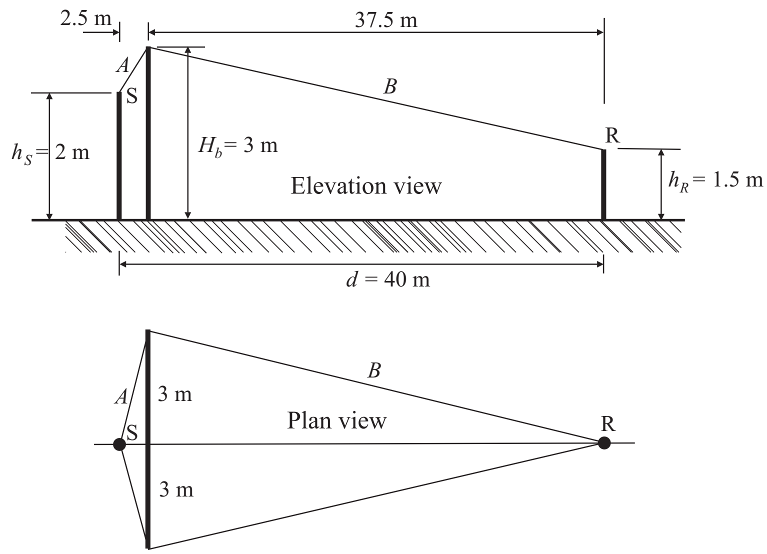


**FIGURE 4.6** Wall arrangement for Problem 8.

reduction for the 63 Hz band is:

$$\text{NR (63 Hz)} = -10\log_{10}\left[10^{-5.77/10} + 2\times 10^{-11.52/10}\right] = 3.92 \text{ (dB)}.$$

**TABLE 4.3** Results for Problem 8(a)

| | Octave band centre frequency (Hz) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 63 | 125 | 250 | 500 | 1000 | 2000 | 4000 | A-wt |
| Octave band sound pressure level (dB re 20 µPa) | 75.0 | 64.0 | 55.0 | 50 | 48.0 | 46.0 | 42.0 | 55.6 |
| $\lambda = c/f$ m | 5.444 | 2.744 | 1.372 | 0.686 | 0.343 | 0.172 | 0.0858 | |
| Wall noise reduction, $D_1$ (dB) | 5.77 | 6.58 | 7.85 | 9.63 | 11.87 | 14.43 | 17.20 | |
| Wall noise reduction, $D_2 = D_3$ (dB) | 11.52 | 14.02 | 16.76 | 19.63 | 22.57 | 25.55 | 28.54 | |
| Overall wall noise reduction, $A_b$ (dB) | 3.92 | 5.24 | 6.86 | 8.84 | 11.19 | 13.81 | 16.61 | |
| New sound pressure level (dB re 20 µPa) | 71.1 | 58.8 | 48.1 | 41.2 | 36.8 | 32.2 | 25.4 | 48.6 |

The A-weighted level is calculated by summing the octave band levels that have been corrected with the A-weighting. Using Equation (1.74), the overall A-weighted level, $L_{p,\text{tot}}$, for the existing noise levels is:

$$L_{p,\text{tot}} = 10\log_{10}\left[10^{(75-26.2)/10} + 10^{(64-16.1)/10} + 10^{(55-8.6)/10} + 10^{(50-3.2)/10} + 10^{48/10} + 10^{(46+1.2)/10} + 10^{(42+1)/10}\right] = 55.6 \text{ (dBA re 20 µPa)}.$$

A similar procedure is followed to obtain the overall A-weighted level after the wall is in place.

(b) The effect of adding a second wall separated by 2 m from the existing wall, on the receiver side of the existing wall, is calculated using the same procedure as in the solution to part (a), except that the quantity $e$, is changed from 0.0 to 2.0.

The quantity $B_1$ becomes, $B_1 = \sqrt{(3-1.5)^2 + (40-2.5-2)^2} = 35.532$ m.

and the quantity, $B_2$ becomes, $B_2 = \sqrt{(3)^2 + (40-2.5-2)^2} = 35.627$ m.

$K_{\text{met}}(\text{top}) = 0.955$.

The results are shown in Table 4.4 below.

**TABLE 4.4** Results for Problem 8(b) (double wall)

| | Octave band centre frequency (Hz) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 63 | 125 | 250 | 500 | 1000 | 2000 | 4000 | A-wt |
| Octave band sound pressure level (dB re 20 µPa) | 75.0 | 64.0 | 55.0 | 50 | 48.0 | 46.0 | 42.0 | 55.6 |
| $\lambda = c/f$ m | 5.444 | 2.744 | 1.372 | 0.686 | 0.343 | 0.172 | 0.0858 | |
| $C_3$ | 1.004 | 1.014 | 1.055 | 1.204 | 1.624 | 2.289 | 2.758 | |
| Wall noise reduction, $D_1$ (dB) | 5.79 | 6.61 | 7.99 | 10.22 | 13.67 | 17.79 | 21.48 | |
| Wall noise reduction, $D_2 = D_3$ (dB) | 11.55 | 14.09 | 17.00 | 20.43 | 24.67 | 29.14 | 32.95 | |
| Overall wall noise reduction, $A_b$ (dB) | 3.94 | 5.29 | 7.02 | 9.46 | 13.03 | 17.20 | 20.90 | |
| New sound pressure level (dB re 20 µPa) | 71.1 | 58.7 | 48.0 | 40.5 | 35.0 | 28.8 | 21.1 | 48.3 |

The effect of increasing the wall height by 1 m to 4 m, is calculated using the same procedure as in the solution to part (a).

The new variable values for use with Equation (4.43) are:

$C_2(\text{top}) = 20$, $C_2(\text{sides}) = 40$, $C_3 = 0$ (single wall).

$A_1 = \sqrt{2.5^2 + 2^2} = 3.202$ m, $A_2 = A_3 = \sqrt{2.5^2 + 3^2} = 3.905$ m.

$B_1 = \sqrt{2.5^2 + 37.5^2} = 37.583$ m, $B_2 = B_3 = \sqrt{3^2 + 37.5^2} = 37.620$ m.

$d_{\text{SR}} = 40$ m.

Thus, $\Delta z_1 = A_1 + B_1 - d_{\text{SR}} = 0.785$, $\Delta z_2 = \Delta z_3 = A_2 + B_2 - d_{\text{SR}} = 1.525$ m.

From Equation (4.51):

$$K_{\text{met}}(\text{top}) = \exp\left[-\frac{1}{2000}\sqrt{\frac{3.202 \times 37.583 \times 40.0}{2(3.202 + 37.583 - 40.0)}}\right] = 1.525.$$

$K_{\text{met}}(\text{sides}) = 1.0$ (not calculated for side diffraction).

The results are shown in Table 4.5 below.

**TABLE 4.5** Results for Problem 8(b) (1 m higher wall)

| | Octave band centre frequency (Hz) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 63 | 125 | 250 | 500 | 1000 | 2000 | 4000 | A-wt |
| Octave band sound pressure level (dB re 20 µPa) | 75.0 | 64.0 | 55.0 | 50 | 48.0 | 46.0 | 42.0 | 55.6 |
| $\lambda = c/f$ m | 5.444 | 2.744 | 1.372 | 0.686 | 0.343 | 0.172 | 0.0858 | |
| Wall noise reduction, $D_1$ | 7.64 | 9.33 | 11.50 | 14.02 | 16.77 | 19.64 | 22.58 | |
| Wall noise reduction, $D_2 = D_3$ | 11.52 | 14.02 | 16.76 | 19.63 | 22.57 | 25.55 | 28.54 | |
| Overall wall noise reduction, $A_b$ | 5.04 | 7.08 | 9.47 | 12.12 | 14.93 | 17.84 | 20.80 | |
| New sound pressure level (dB re 20 µPa) | 70.0 | 56.9 | 45.5 | 37.9 | 33.1 | 28.2 | 21.2 | 46.7 |

.

Thus it is more beneficial to add 1 m to the height of the wall, resulting in an additional 1.9 dBA attenuation.

# 5

# Solutions to Additional Problems in Chapter 5

## Problem 1

(a) Set the origin, $x = 0$, at the surface of the sample. Then the entrance of the tube is at $x = -2.0$. As the origin is at the surface of the sample, the incident wave will be travelling in the positive $x$-direction. Assuming a phase shift between the incident and reflected waves of $\beta$ at $x = 0$, the incident wave and reflected wave pressures may be written using Equation (1.55) as:

$$p_i = A\mathrm{e}^{\mathrm{j}(\omega t - kx)} \quad \text{and} \quad p_r = B\mathrm{e}^{\mathrm{j}(\omega t + kx + \beta)}$$

The total pressure is thus:

$$p_T = A\mathrm{e}^{\mathrm{j}(\omega t - kx)} + B\mathrm{e}^{\mathrm{j}(\omega t + kx + \beta)}$$

The total particle velocity can be calculated using Equations (1.2) and (1.3) in the text as:

$$u_T = \frac{1}{\rho c}\left(A\mathrm{e}^{\mathrm{j}(\omega t - kx)} - B\mathrm{e}^{\mathrm{j}(\omega t + kx + \beta)}\right)$$

The pressure reflection coefficient, $R_p$, is defined as $p_r/p_i$. Thus:

$$R_p = (B/A)\mathrm{e}^{\mathrm{j}\beta} = 0.75 + 0.3\mathrm{j}$$

The reflection coefficient amplitude is $B/A$ and the phase is $\beta$. Thus, $B/A = \sqrt{0.75^2 + 0.3^2} = 0.8078$ and $\beta = \arctan(0.3/0.75) = 0.3805$ radians. Thus, the specific acoustic impedance at any point in the tube may be written as:

$$Z_s = \frac{p_T}{u_T} = \rho c \frac{A\mathrm{e}^{-\mathrm{j}kx} + B\mathrm{e}^{\mathrm{j}kx + \mathrm{j}\beta}}{A\mathrm{e}^{-\mathrm{j}kx} - B\mathrm{e}^{\mathrm{j}kx + \mathrm{j}\beta}} = \rho c \frac{A + B\mathrm{e}^{\mathrm{j}(2kx + \beta)}}{A - B\mathrm{e}^{\mathrm{j}(2kx + \beta)}}$$

$$= \rho c \frac{A/B + \cos(2kx + \beta) + \mathrm{j}\sin(2kx + \beta)}{A/B - \cos(2kx + \beta) - \mathrm{j}\sin(2kx + \beta)}$$

$k = \omega/c = 2\pi \times 100/343 = 1.832$ ($\mathrm{m}^{-1}$) and $x = -2$ (m).
Thus, $2kx + \beta = -7.32733 + 0.3805 = -6.947$ radians.
$\cos(2kx + \beta) = 0.7878$, $\sin(2kx + \beta) = -0.6160$ and $A/B = 1.238$. Thus,

$$
\begin{aligned}
Z_s &= 413.6\left[\frac{1.2380 + 0.7878 - \mathrm{j}0.6160}{1.2380 - 0.7878 + \mathrm{j}0.6160}\right] = 413.6\left[\frac{2.0258 - \mathrm{j}0.6160}{0.4502 + \mathrm{j}0.6160}\right]\\
&= 413.6\left[\frac{(2.0258 - \mathrm{j}0.6160)(0.4502 - \mathrm{j}0.6160)}{(0.4502 + \mathrm{j}0.6160)(0.4502 - \mathrm{j}0.6160)}\right] = 413.6\left[\frac{0.6371 - \mathrm{j}1.5252}{0.5821}\right]\\
&= 453 - \mathrm{j}1080 \text{ (Rayls) or } (\mathrm{kg\,m^{-2}\,s^{-1}}) = \text{impedance looking into the open end.}
\end{aligned}
$$

(b) The normal incidence absorption coefficient is defined in terms of the reflection coefficient at the surface of the sample as:
$\alpha_n = 1 - |R_p|^2 = 1 - (0.75^2 + 0.3^2) = 0.35$

(c) The statistical absorption coefficient is calculated from the normal specific impedance at the surface of the sample ($x = 0$) using Equations (5.39) and (5.45) in the textbook. We can use the preceding equation with $x = 0$, $\cos\beta = \cos(0.3805^c) = 0.9285$ and $\sin\beta = \sin(0.3805^c) = 0.3714$. Thus:

$$
\begin{aligned}
\frac{Z_N}{\rho c} = \frac{Z_s}{\rho c}\bigg|_{x=0} &= \frac{1.2380 + 0.9285 + \mathrm{j}0.3714}{1.2380 - 0.9285 - \mathrm{j}0.3714} = \frac{2.1665 + \mathrm{j}0.3714}{0.3095 - \mathrm{j}0.3714}\\
&= \frac{(2.1665 + \mathrm{j}0.3714)(0.3095 + \mathrm{j}0.3714)}{(0.3095 - \mathrm{j}0.3714)(0.3095 + \mathrm{j}0.3714)} = \frac{0.5326 + \mathrm{j}0.9196}{0.2337}\\
&= 2.2787 + \mathrm{j}3.9345 = 4.5467\mathrm{e}^{\mathrm{j}1.0458^c}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{st} &= \left\{\frac{8\cos(1.0458^c)}{4.5467}\right\}\left\{1.0 - \left[\frac{\cos(1.0458)}{4.5467}\right]\log_\mathrm{e}\big(1.0 + 2.0\times 4.5467\cos(1.0458)\right.\\
&\qquad \left. + 4.5467^2\big) + \left[\frac{\cos(2\times 1.0458)}{4.5467\times\sin(1.0458)}\right]\tan^{-1}\left[\frac{4.5467\sin(1.0458)}{1 + 4.5467\cos(1.0458)}\right]\right\}\\
&= 0.47
\end{aligned}
$$

## Problem 2

(a) To simplify the algebra, assume that the tube is horizontal, with the left end at $x = 0$, containing the sample of material whose absorption coefficient is to be determined, as shown in Figure 5.1.

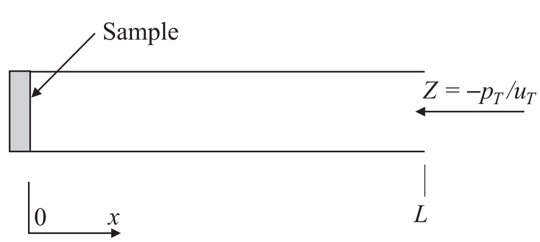


**FIGURE 5.1** Arrangement for Problem 2.

As the origin is at the left end of the tube, the incident wave will be travelling in the negative $x$ direction. Assuming a phase shift between the incident and reflected waves of $\beta$ at $x = 0$, the incident wave and reflected wave pressures may be written, using Equation (1.36), as:

$$
p_i = A\mathrm{e}^{\mathrm{j}(\omega t + kx)} \quad \text{and} \quad p_r = B\mathrm{e}^{\mathrm{j}(\omega t - kx + \beta)}
$$

The total pressure is thus:

$$p_T = A\mathrm{e}^{\mathrm{j}(\omega t+kx)} + B\mathrm{e}^{\mathrm{j}(\omega t-kx+\beta)}$$

By inspection, the maximum pressure will occur when $\beta = 2kx$, and the minimum will occur when $\beta = 2kx + \pi$. Thus:
$p_{\max} = \mathrm{e}^{\mathrm{j}kx}(A+B)$
$p_{\min} = \mathrm{e}^{\mathrm{j}kx}(A-B)$
and the ratio of maximum to minimum pressures is $(A+B)/(A-B)$

The standing wave ratio, $L_0$, is defined as:

$$10^{L_0/20} = \frac{A+B}{A-B}$$

Thus, the ratio (B/A) is

$$\frac{B}{A} = \left[\frac{10^{L_0/20}-1}{10^{L_0/20}+1}\right]$$

(b) The difference in the sound pressure levels of the two minima are a result of losses in the tube due to interaction of the sound wave with the walls of the tube and absorption by the air in the tube. Thus it is necessary to extrapolate the levels of the two minima back to the location of the surface of the test sample on a plot of dB vs location in the tube as shown in Figure 5.2.

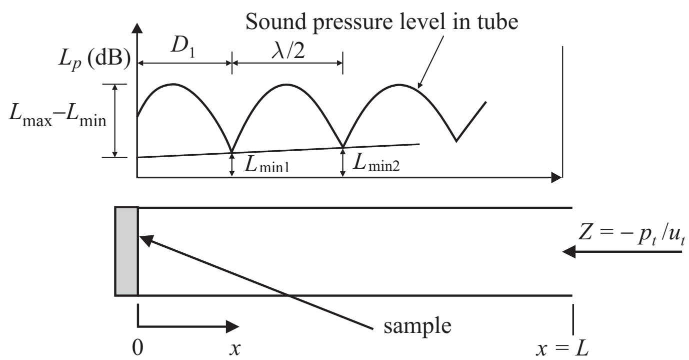


**FIGURE 5.2** Sound pressure level in the tube for Problem 2.

The spacing between adjacent minima is half of a wavelength of sound at the tonal input frequency Using similar triangles, the minimum sound pressure level to be used in the calculation of $L_0$ is calculated as $\dfrac{L_{\min 1}-L_{\min}}{D_1} = \dfrac{L_{\min 2}-L_{\min}}{D_1+\lambda/2}$. Thus,

$L_{\min} = L_{\min 1} - \dfrac{2D_1}{\lambda}(L_{\min 2}-L_{\min 1}) = 82 - \dfrac{0.1\times 2}{343/500} = 81.71$ (dB),
which gives, $L_0 = 90 - 81.71 = 8.29$ (dB).
By definition, the amplitude of the pressure reflection coefficient squared is $|R_p|^2 = (B/A)^2$, which can be written in terms of $L_0$ (= 8.29), using Equation (5.30), as:

$$|R_p|^2 = \left[\frac{10^{L_0/20}-1}{10^{L_0/20}+1}\right]^2$$

The normal incidence absorption coefficient is defined as $\alpha_n = 1 - |R_p|^2$, so

$$\alpha_n = 1 - \left[\frac{10^{8.29/20} - 1}{10^{8.29/20} + 1}\right]^2 = 0.80$$

(c) The losses in the tube are defined by Equation (5.21). Thus:
$a = 0.19137\sqrt{500}/(343 \times 0.025) = 0.5$ dB/m.

## Problem 3

As given in the problem, the tube is assumed to be horizontal with the left end at $x = 0$ containing the sample of material whose impedance is to be determined, as shown Figure 5.1.

As the origin is at the left end of the tube, the incident wave will be travelling in the negative $x$-direction. Assuming a phase shift between the incident and reflected waves of $\beta$ at $x = 0$, the incident wave and reflected wave pressures may be written, using Equation (1.36), as:

$$p_i = A\mathrm{e}^{\mathrm{j}(\omega t + kx)} \quad \text{and} \quad p_r = B\mathrm{e}^{\mathrm{j}(\omega t - kx + \beta)}$$

The total pressure is thus:

$$p_T = A\mathrm{e}^{\mathrm{j}(\omega t + kx)} + B\mathrm{e}^{\mathrm{j}(\omega t - kx + \beta)}$$

At the surface of the sample, the pressure amplitude reflection coefficient is thus $R_p = (B/A)\mathrm{e}^{\mathrm{j}\beta}$ and $B = (R_p A)\mathrm{e}^{-\mathrm{j}\beta}$. Thus the total pressure at any location, $x$, in the tube may be written as:

$$p_T = A\left(\mathrm{e}^{\mathrm{j}(\omega t + kx)} + R_p\mathrm{e}^{\mathrm{j}(\omega t - kx)}\right)$$

## Problem 4

From Equation (5.65):

$$\text{NRC} = \frac{\bar{\alpha}_{250} + \bar{\alpha}_{500} + \bar{\alpha}_{1000} + \bar{\alpha}_{2000}}{4} = \frac{0.7 + 0.8 + 1.0 + 1.0}{4} = 0.88$$

## Problem 5

Following Equation (5.66) in the text:

$$\bar{\alpha} = \frac{\sum S_i \bar{\alpha}_i}{\sum S_i}$$

$$= \frac{2 \times 6.84 \times 5.565 \times 0.02 + 2 \times 5.565 \times 4.72 \times 0.05 + 2 \times 6.84 \times 4.72 \times 0.06}{2 \times 6.845.565 + 2 \times 5.565 \times 4.72 + 2 \times 6.84 \times 4.72}$$

$$= 8.023/193.2 = 0.042$$

## Problem 6

(a) From Problem 2, the minimum sound pressure level to be used in the calculation of $L_0$ is:
$L_{\min} = L_{\min 1} - \dfrac{2D_1}{\lambda}(L_{\min 2} - L_{\min 1}) = 90 - \dfrac{0.15 \times 2}{343/315}(91.5 - 90) = 89.59$ (dB),

which gives, $L_0 = 99 - 89.6 = 9.41$ (dB).
Using Equation (5.30) in the textbook, the amplitude of the pressure reflection coefficient squared, $|R_p|^2 = (B/A)^2$, can be written in terms of $L_0$ (= 9.41) as:

$$|R_p|^2 = \left[\frac{10^{L_0/20} - 1}{10^{L_0/20} + 1}\right]^2$$

The normal incidence sound absorption coefficient is defined as $\alpha_n = 1 - |R_p|^2$, so

$$\alpha_n = 1 - \left[\frac{10^{9.41/20} - 1}{10^{9.41/20} + 1}\right]^2 = 0.756$$

(b) The sound intensity, $I_i$, of the incident wave is given by Equation (1.57) as:
$I_i = \langle p_i^2 \rangle/(\rho c) = \hat{p}_i^2/(2\rho c)$.
The incident sound pressure amplitude, $\hat{p}_i$ is given by: $\hat{p}_i = A$, where $A$ is defined in Equation (5.23) in the textbook. The value of $A$ can be calculated from the maximum and minimum sound pressure amplitudes (see Equation (5.27) in the textbook), which are $(A+B)$ and $(A-B)$, respectively, The maxima and minima in the standing wave occur at locations such that $\cos(kx) = 1$ and $\sin kx = 0$. Thus:
$A = (1/2)(\hat{p}_{\max} + \hat{p}_{\min}) = (\sqrt{2}/2) \times 20 \times 10^{-6} \times (10^{99/20} + 10^{89.59/20}) = 1.687$ (Pa).
Thus, $I_i = 1.6870^2/(2 \times 413.6) = 3.44\,(\text{mW/m}^2)$.

(c) The sound intensity, $I_r$, of the wave propagating away from the surface of the sample is given by:
$I_r = I_i(1 - \alpha)$, where $\alpha$ is the normal incidence absorption coefficient. Thus,
$I_r = 3.44(1 - 0.756) = 0.840\,(\text{mW/m}^2)$.

(d) The acoustic power dissipated by the sample is the product of the sound intensity absorbed by the sample multiplied by the sample surface area. Thus:
$W_a = I_i \alpha_n S = 3.44 \times 10^{-3} \times 0.756 \times (\pi \times 0.1^2/4) = 20.4\,(\mu\text{W})$.

(e) The acoustic power dissipated in the tube due to air absorption and absorption by the walls of the tube can be calculated using Equation (5.21) in the textbook and taking into account the fact that sound is travelling in both directions. From Equations (1.57) and (1.61), the power dissipated during travel in one direction is given by:
$W_{\text{tube}} = (p_e^2 - p_r^2)S/(\rho c) = p_r^2[(p_e/p_r)^2 - 1]S/(\rho c)$,
where $p_e$ is the acoustic pressure of the incident wave at the loudspeaker and $p_r$ is the acoustic pressure of the wave reflected from the surface of the sample. From Equation (5.27):
$B = (1/2)(\hat{p}_{\max} - \hat{p}_{\min}) = (\sqrt{2}/2) \times 20 \times 10^{-6} \times (10^{99/20} - 10^{89.59/20}) = 0.834$ Pa.
Alternatively, the amplitude, $\hat{p}_r = B$, of the reflected wave is related to the reflected sound intensity, $I_r$, by $\hat{p}_r = B = \sqrt{2\rho c I_r} = \sqrt{2 \times 413.6 \times 0.840/1000} = 0.834$ Pa.
The acoustic pressure, $p_e$, at the end of the tube adjacent to the loudspeaker source is calculated using the value of $p_r$ and Equations (5.20) and (5.21) in the textbook as follows.
$p_e^2 = p_r^2 \times 10^{(L_{pe} - L_{pr})/10} = p_r^2 \times 10^{(aL)/10}$.
Thus, $(p_e/p_r)^2 = 10^{(aL)/10}$, where $L$ is the length of the tube.
The quantity, $a$ is given by Equation (5.21) in the textbook as:
$a = 0.19137\sqrt{f}/[c \times (4S/P_D)] = 0.19137\sqrt{315}/[343 \times 0.1] = 0.099$
Thus, $(p_e/p_r)^2 = 10^{(0.099 \times 2.5)/10} = 1.0587$
and the total power dissipated in the tube (due to waves travelling in both directions is $2W_{\text{tube}} = 2 \times 0.834^2[1.0587 - 1](\pi \times 0.1^2/4)/413.6 = 1.55\,\mu\text{W}$.

(f) The minimum continuous electric power rating, $P_W$, of the loudspeaker will be equal to the acoustic power dissipated in the sample and tube divided by the electrical to

acoustic conversion efficiency (approximately 1%). Thus:
$P_W = (20.5 + 1.6)/0.01 = 2.2$ (mW).
However, we would usually only wish to operate the speaker at 10% of its power rating to minimise harmonic distortion, so we would require a speaker to be rated for at least 220 mW of continuous power.

## Problem 7

The statistical absorption coefficient, $\alpha_{st}$, of a construction is given in terms of the normal surface impedance, $Z_N = \rho c \xi \mathrm{e}^{\mathrm{j}\psi}$, of the structure by Equation (5.45) in the textbook. For sound absorbing material covered with a perforated panel, the normal surface impedance is given by Equation (5.51) in the textbook. In the case considered here, there is no flow so $M = 0$. For maximum absorption at 500 Hz, Equations (5.45) and (5.51) in the textbook could be used to calculate the statistical absorption coefficient at 500 Hz and the percentage open area that gives maximum absorption at 500 Hz could be found by trial and error. We shall call this method 1. Alternatively, Equation (5.53) or (5.54) in the textbook could be used. We will call this Method 2. As the porous material fills the entire backing cavity, we assume isothermal propagation so we multiply the adiabatic speed of sound by 0.85. Thus, the quantity, $f_{\max} L_{\text{tot}}/c = 500 \times 0.103/(343 \times 0.85) = 0.18$, which is greater than 0.1, so Equation (5.54) should be used. However, we will also check the result by solving Equation (5.53) to observe what the error would be.

### Method 1

First we calculate the characteristic impedance of the porous material. The material flow resistivity is given as $10^4$ MKS Rayls/m. Thus from Equation (5.15) in the textbook, the dimensionless quantity, $X = 1.206 \times 500/10^4 = 6.03 \times 10^{-2}$. Thus from Equation (5.16) in the textbook, the normalised characteristic impedance is:

$$\frac{Z_m}{\rho c} = \left[1 + 0.0571 \times (6.03 \times 10^{-2})^{-0.754} - \mathrm{j}0.087 \times (6.03 \times 10^{-2})^{-0.732}\right]$$
$$= 1.4745 - \mathrm{j}0.6797$$

From Equation (5.17), the normalised wavenumber in the porous material is:

$$\frac{k_m}{k} = \left[1 + 0.0978 \times (6.03 \times 10^{-2})^{-0.700} - \mathrm{j}0.189 \times (6.03 \times 10^{-2})^{-0.595}\right]$$
$$= 1.0978 - \mathrm{j}1.005$$

where $k = 2\pi \times 500/(343 \times 0.85) = 10.776$ m$^{-1}$. As the porous material is rigidly backed, the normal impedance, $Z_N$, is given by Equation (5.49) in the textbook. Thus:

$$\frac{Z_N}{\rho c} = -\mathrm{j}\frac{(1.475 - \mathrm{j}0.6797)}{\tan\{[2\pi \times 500/(343 \times 0.85)][1.0978 - \mathrm{j}1.005]0.1\}} = 1.4377 - \mathrm{j}0.7711$$

To calculate the impedance effect of the perforated sheet, we need to calculate the hole spacing, $q$, the effective length, $\ell_e$, of the holes, the mass per unit area, $m$, of the perforated sheet and the acoustic resistance, $R_A$ of the holes. If we assume a staggered hole arrangement, as shown in Figure 5.14 in the textbook and refer to the solution of Example 5.9 in the textbook, we have:

$$q = \sqrt{\frac{\pi \times d_h^2}{2 \times (P/100)\sqrt{3}}} = \sqrt{\frac{\pi \times 0.003^2}{2 \times (P/100)\sqrt{3}}} = 0.0286\sqrt{1/P} \text{ (m)}$$

The effective length of the holes in the perforated sheet is given by Equation (5.52) in the textbook as:

$$\ell_e = 0.003 + \left[\frac{16 \times 0.0015}{3\pi}(1 - 0.43 \times 0.0015\sqrt{P}/0.286)\right] = 0.0055 - 5.74 \times 10^{-6}\sqrt{P}$$

The mass per unit area of the perforated sheet = $\rho_m h$(1−fraction open area). Thus:
$m = 7850 \times 0.003(100 - P)/100 = 23.55(100 - P)/100$ kg/m$^2$

From Equation (5.51) in the textbook:

$$\frac{Z_{NP}}{\rho c} = R + \mathrm{j}X = 1.4377 - \mathrm{j}0.7711$$

$$+ \frac{(100/P)\left[\mathrm{j}\tan(10.775 \times \ell_e) + R_A \times \pi \times 0.003^2/(4 \times 413.6)\right]}{1 + \dfrac{100}{2\pi \times 500(23.55(100 - P)/100)P}\left[413.6\tan(10.775 \times \ell_e) + R_A \times \pi \times 0.003^2/4\right]}$$

$$= \xi \mathrm{e}^{\mathrm{j}\psi}, \text{ where } \xi = \sqrt{R^2 + X^2} \text{ and } \psi = \arctan X/R.$$

The resistance term, $R_A$ is defined in Equation (8.21) and (8.22) and is given by:

$$\frac{R_A S}{\rho c} = \frac{10.7755 \times \sqrt{2 \times 1.84 \times 10^{-5}/(1.206 \times 2\pi \times 500)} \times \pi \times 0.003 \times 0.003}{2\pi \times 0.003^2/4} \times$$

$$\times \left[1 + 0.4 \times \sqrt{\frac{5}{3 \times 1.4}}\,\right]$$

$$+ 0.288 \times 10.7755 \times \sqrt{2 \times 1.84 \times 10^{-5}/(1.206 \times 2\pi \times 500)} \times \log_{10}\left[\frac{0.003^2}{0.0015^2}\right] + 0 + 0$$

$$= 0.003235$$

We can now use Equation (5.45) in the textbook to find the statistical absorption coefficient for a given % open area of the perforated sheet. We can use trial and error (with an excel spreadsheet) with this equation to find the value of % open areas, $P$ that will give maximum absorption at 500 Hz. The result for optimum $P$ is 11.4%.

**Method 2**

Using Equation (5.53) in the textbook gives:

$$200 = \frac{343 \times 0.85}{2\pi}\left[\frac{P/100}{0.103[0.003 + 0.85 \times 0.003(1 - 0.22 \times 0.003\sqrt{P}/0.0286)]}\right]^{1/2}$$

which can be rewritten as:

$$\frac{P}{10.3\left[0.003 + 0.85 \times 0.003\left(1 - \dfrac{0.22 \times 0.003\sqrt{P}}{0.0286}\right)\right]} = \left(\frac{500 \times 2\pi}{343 \times 0.85}\right)^2$$

Solving by trial and error gives $P = 6.5\%$.

Using Equation (5.54) gives:

$$2\pi \times 100 \times 0.103/(343 \times 0.85)\tan[2\pi \times 100 \times 0.103/(343 \times 0.85)]$$
$$= \frac{P \times 0.103/100}{0.003 + 0.85 \times 0.003(1 - 0.22 \times 0.003 \times 0.0286/\sqrt{P})}$$

which can be solved by trial and error to give $P = 11.7\%$. This is very close to that obtained using method 1.

If we used the adiabatic speed of sound instead (due to 500 Hz being classified as a marginally low frequency), the results for method 1, method 2 (approx) and method 2 (accurate), respectively would be 6.4%, 4.7% and 6.2%, respectively. However, any value of % open area between 6% and 11% would give almost the same statistical absorption coefficient.

## Problem 8

The room volume, $V = 30 \times 20 \times 3 = 1800$ m$^3$.
The total amount of additional absorption needed to achieve the desired reverberation times can be calculated using Figures 5.15 and 5.16 and Equation (5.63) in the textbook. The room surface area is $2 \times (30 \times 20 + 30 \times 3 + 20 \times 3) = 1500$ m$^2$. Excluding the floor, the area is 900 m$^2$. The average room absorption coefficient for the existing room may be calculated from the existing reverberation time for each octave band using Equation (5.61) in the textbook. Thus:
$\bar{\alpha} = \dfrac{55.25 \times 1800}{1500 \times 343 \times T_{60}}$. These values are included in Table 5.1, line 2.
Assuming that the contribution of air absorption is the same before and after addition of the absorbing material, the additional absorption (in m$^2$) required of the room treatment is given by Equation (5.63) in the textbook as:

$$S\bar{\alpha}_{\text{additional}} = \frac{55.25 \times 1800}{343}\left[\frac{1}{T_{60\text{orig}}} - \frac{1}{T_{60\text{new}}}\right]$$

These values are included in Table 5.1, line 4.

Note that the sound absorbing material will be covering some room surfaces that were contributing to the original absorption and this needs to be taken into account by adjusting the absorption coefficient of the material. The effective absorption coefficient of the added material or wall panel is then the difference between the actual absorption coefficient and the existing absorption coefficient (see Table 5.1, line 6 below). The aim is to add sufficient material to satisfy the required additional absorption requirement at the two highest frequencies and then use panel absorbers to make up deficiencies in the 4 lower frequencies. Thus, for an effective absorption coefficient of 0.758 at the highest 2 frequencies, the required area of material is $362.4/0.758 = 478$ m$^2$. The amount of absorption provided by this area of sound absorbing material is obtained for each octave band centre frequency by multiplying this area of added material by the difference between the material absorption coefficient and the room surface average absorption coefficient. These results are in Table 5.1, line 7 below. The amount of absorption required of the panel absorbers is equal to the total amount of absorption required minus the contribution of the rockwool material. These results are in Table 5.1, line 8 below.

In selecting which curve in Figure 5.15 in the textbook that we should use in the design, we look at the ratio of absorption requirements at twice and 4 times the design frequency and see that the requirement at twice the design frequency is 0.77 times that required at the design frequency and the requirement at 4 times the design frequency is 0.56 times that required at the design frequency. The curve that is closest to this requirement is curve A. However, this curve requires a very deep backing cavity which may be impractical so we will choose curve C. We would only choose curves A or B if we were short of wall space on which to mount the absorbers. An additional consideration is the requirement to change the design frequency by a factor calculated using Equation (5.58) in the textbook. If we choose curve C we would require a panel with a surface mass of approximately 1.5 kg/m$^2$. Substituting this value into Equation (5.58) in the textbook and assuming a panel size of 1 m $\times$ 2 m, the new design frequency will be approximately 100 Hz. From Curve C at 100 Hz in Figure 5.16, we obtain a panel absorber with a cavity depth of 200 mm and a mass of 1.6 kg/m$^2$. There also needs to be a rockwool of fibreglass blanket attached to the wall behind the panel. A thickness of 25 mm would be adequate. From Figure 5.15, we obtain the absorption coefficients in line 9 of Table 5.1 below.

The effective absorption coefficient for the panel is calculated using the existing absorption coefficient and the panel absorption coefficient in the same way as already done for the rockwool material. However, in this case the effective absorption is negative at the higher frequencies, which indicates that the panel will be less absorptive than the existing room surfaces at these frequencies, so the area of sound absorptive material will have to be increased to account for this.

Based on the absorption requirement at 125 Hz, the required panel area is $S_p = 185.9/0.78 = 240$ m$^2$. This results in the added absorption due to the added panels shown in line 10 of Table 5.1 below. To achieve the required absorption at all frequencies is not possible, so a compromise is necessary, whereby the absorption will be greater than required at some frequencies and less than required at other frequencies.

If it is more important to achieve the required absorption or greater at the mid and higher frequencies than it is at lower frequencies, then we need to add more rockwool material and change the panel design to be optimal at 250 Hz rather than 125 Hz. We could begin by using more rockwool to achieve the required absorption at 1000 Hz and at the same time increase the panel area by 20%. This would require an area of rockwool material of $(402.7 + 1.2 \times 31)/0.685 = 642$ m$^2$. The new additional absorption due to this area of rockwool plus 20% increase in area of panels is shown in line 11 of Table 5.1 below. This is calculated by multiplying line 6 by 440 and adding the result to line 10 multiplied by 1.2. With this design, the reverberation time will be too long at 250 Hz and too short at 2000 Hz and 4000 Hz.

One way of improving the 250 Hz problem would be to use a second panel design for which the maximum absorption is at 250 Hz. Then the mix of panels with a maximum absorption at 125 Hz and a maximum at 250 Hz could be optimised. The higher frequency problem could be partially alleviated by using panels with maximum absorption at 500 Hz. This problem can be solved as an optimisation problem, assuming that it is equally acceptable for any frequency to be deficient in absorption or to have excessive absorption of the same percentage and that the total squared error in absorption ([actual absorption − required absorption, $S\alpha$]$^2$) is minimised. Thus the requirement would be to minimise:

$$(S_1\alpha_1 + S_2\alpha_2 + S_3\alpha_1 + S_4\alpha_2 - S\bar{\alpha})^2_{125\text{ Hz}} + (S_1\alpha_1 + S_2\alpha_2 + S_3\alpha_1 + S_4\alpha_2 - S\bar{\alpha})^2_{250\text{ Hz}}$$
$$+(S_1\alpha_1 + S_2\alpha_2 + S_3\alpha_1 + S_4\alpha_2 - S\bar{\alpha})^2_{500\text{ Hz}} + (S_1\alpha_1 + S_2\alpha_2 + S_3\alpha_1 + S_4\alpha_2 - S\bar{\alpha})^2_{1000\text{ Hz}}$$
$$+(S_1\alpha_1 + S_2\alpha_2 + S_3\alpha_1 + S_4\alpha_2 - S\bar{\alpha})^2_{2000\text{ Hz}} + (S_1\alpha_1 + S_2\alpha_2 + S_3\alpha_1 + S_4\alpha_2 - S\bar{\alpha})^2_{4000\text{ Hz}}$$

where the subscripts 1, 2, 3 and 4 refer, respectively, to the rockwool material, the panel optimised at 125 Hz, the panel optimised at 250 Hz and the panel optimised at 500 Hz. The values of $\alpha$ are effective absorption coefficients of the respective constructions. The optimisation problem is to find optimal values of the areas $S_1$, $S_2$, $S_3$ and $S_4$ that will minimise the above expression. To do this we need values for each $\alpha$. We already have values at each frequency for $\bar{\alpha}$, $\alpha_1$ and $\alpha_2$. Before we can proceed we need values for $\alpha_3$ and $\alpha_4$ for each frequency (corresponding to panel absorbers optimised at 250 Hz and 500 Hz, respectively).

Using the excel solver function in the data analysis tab to solve the optimisation problems for the three cases gives the following results (see also the last 3 lines of Table 5.2 below and compare with the second line in the table to see how close the results are to the requirement). In the following list, the total square error is the sum of the squared differences (for octave band centre frequencies between 125 Hz and 4000 Hz) between the actual $S\alpha$ achieved and the requirement (line 2 in Table 5.2 below). All panels are designed for curve C in Figure 5.15 in the textbook and all have 25 mm of rockwool fixed to the wall in the cavity behind the panel.

1. Panel optimised at 125 Hz plus rockwool blanket
   (a) Required panel area = 270 m$^2$, backing cavity depth = 200 mm and panel surface mass = 1.6 kg/m$^2$
   (b) Required rockwool area = 574 m$^2$
   (c) Total square error = 7768 m$^4$
2. Panel optimised for 125 Hz plus panel optimised at 250 Hz plus rockwool blanket
   (a) 125 Hz panel area = 213 m$^2$, backing cavity depth = 200 mm and panel surface mass = 1.6 kg/m$^2$

(b) 250 Hz panel area = 108 $m^2$, backing cavity depth = 105 mm and panel surface mass = 0.72 $kg/m^2$

(c) Required rockwool area = 564 $m^2$

(d) Total square error = 1665 $m^4$

3. Panel optimised for 125 Hz plus panel optimised for 250 Hz plus panel optimised for 500 Hz plus rockwool blanket

   (a) 125 Hz panel area = 214 $m^2$, backing cavity depth = 200 mm and panel surface mass = 1.6 $kg/m^2$

   (b) 250 Hz panel area = 101 $m^2$, backing cavity depth = 105 mm and panel surface mass = 0.72 $kg/m^2$

   (c) 500 Hz panel area = 16 $m^2$, backing cavity depth = 52 mm and panel surface mass = 0.42 $kg/m^2$

   (d) Required rockwool area = 561 $m^2$

   (e) Total square error = 1576 $m^4$

It can be seen from the mean square error results that very little is gained by including the 500 Hz panel.

**TABLE 5.1** Results for Problem 8

| | Octave band centre frequency (Hz) | | | | | |
|---|---|---|---|---|---|---|
| | 125 | 250 | 500 | 1000 | 2000 | 4000 |
| Existing reverberation times (s) | 2.0 | 1.6 | 1.2 | 0.9 | 0.8 | 0.8 |
| Existing Sabine absorption coefficient, $S\bar{\alpha}$ | 0.0966 | 0.1208 | 0.1611 | 0.2148 | 0.2416 | 0.2416 |
| Desired reverberation times (s) | 1.0 | 0.8 | 0.5 | 0.4 | 0.4 | 0.4 |
| Additional required $S\alpha_{\text{additional}}$ (m$^2$) | 193.3 | 181.2 | 338.3 | 402.7 | 362.4 | 362.4 |
| Actual rockwool Sabine absorption coefficient | 0.08 | 0.2 | 0.7 | 0.9 | 1.0 | 1.0 |
| Effective rockwool Sabine absorption coefficient | 0.016 | 0.079 | 0.539 | 0.685 | 0.758 | 0.758 |
| $S\alpha$ (m$^2$) provided by 478 m$^2$ of material | 7.4 | 37.9 | 257.6 | 327.5 | 362.5 | 362.5 |
| $S\alpha$ (m$^2$) required of panel absorber | 185.9 | 143.3 | 80.7 | 75.2 | 0 | 0 |
| $\alpha$ corresponding to panel type $C_{125\text{ Hz}}$ | 0.88 | 0.42 | 0.2 | 0.08 | 0.08 | 0.08 |
| Effective $\alpha$ corresponding to panel type $C_{125\text{ Hz}}$ | 0.78 | 0.30 | 0.04 | -0.13 | -0.16 | -0.16 |
| Actual $S\alpha$ (m$^2$) of panel absorber | 187 | 72 | 10 | -31 | -38 | -38 |
| total $S\alpha$ (m$^2$) of first design | 194 | 110 | 268 | 297 | 325 | 325 |
| total $S\alpha$ (m$^2$) of second design | 235 | 137 | 358 | 403 | 441 | 441 |
| $\alpha$ corresponding to panel type $C_{250\text{ Hz}}$ | 0.23 | 0.88 | 0.42 | 0.30 | 0.08 | 0.08 |
| Effective $\alpha$ corresponding to panel type $C_{250\text{ Hz}}$ | 0.13 | 0.76 | 0.26 | 0.09 | -0.16 | -0.16 |
| $\alpha$ corresponding to panel type $C_{500\text{ Hz}}$ | 0.05 | 0.23 | 0.88 | 0.42 | 0.2 | 0.08 |
| Effective $\alpha$ corresponding to panel type $C_{500\text{ Hz}}$ | 0.03 | 0.11 | 0.72 | 0.21 | -0.04 | -0.04 |

**TABLE 5.2** Optimal results (last 3 lines) for various options in Problem 8

| | Octave band centre frequency (Hz) | | | | | |
|---|---|---|---|---|---|---|
| | 125 | 250 | 500 | 1000 | 2000 | 4000 |
| Existing Sabine absorption coefficient | 0.0966 | 0.1208 | 0.1611 | 0.2148 | 0.2416 | 0.2416 |
| Additional required $S\alpha_{\text{additional}}$ ($m^2$) | 193.3 | 181.2 | 338.3 | 402.7 | 362.4 | 362.4 |
| Actual rockwool Sabine absorption coefficient | 0.08 | 0.2 | 0.7 | 0.9 | 1.0 | 1.0 |
| Effective rockwool Sabine absorption coefficient | 0.016 | 0.079 | 0.539 | 0.685 | 0.758 | 0.758 |
| $\alpha$ corresponding to panel type $C_{125 \text{ Hz}}$ | 0.88 | 0.42 | 0.2 | 0.08 | 0.08 | 0.08 |
| Effective $\alpha$ corresponding to panel type $C_{125 \text{ Hz}}$ | 0.78 | 0.30 | 0.04 | -0.13 | -0.16 | -0.16 |
| $\alpha$ corresponding to panel type $C_{250 \text{ Hz}}$ | 0.23 | 0.88 | 0.42 | 0.30 | 0.08 | 0.08 |
| Effective $\alpha$ corresponding to panel type $C_{250 \text{ Hz}}$ | 0.13 | 0.76 | 0.26 | 0.09 | -0.16 | -0.16 |
| $\alpha$ corresponding to panel type $C_{500 \text{ Hz}}$ | 0.05 | 0.23 | 0.88 | 0.42 | 0.2 | 0.08 |
| Effective $\alpha$ corresponding to panel type $C_{500 \text{ Hz}}$ | 0.03 | 0.11 | 0.72 | 0.21 | -0.04 | -0.04 |
| Optimised $S\alpha$ for 125 Hz panel plus rockwool | 220 | 126 | 320 | 360 | 392 | 392 |
| Optimised $S\alpha$ for 125 Hz panel plus 250 Hz panel plus rockwool | 190 | 191 | 341 | 369 | 376 | 376 |
| Optimised $S\alpha$ for 125 Hz panel plus 250 Hz panel plus 500 Hz panel plus rockwool | 189 | 187 | 349 | 369 | 374 | 374 |

## Problem 9

(a) Air absorption should be added to the room surface absorption. The addition due to air absorption is included in Equation (6.9) in the textbook. The room volume, $V = 10 \times 8 \times 4 = 320$ m$^3$. The room surface area, $S = 2(10 \times 8 + 10 \times 4 + 8 \times 4) = 152$ m$^2$. Values of $m$ are obtained from table 4.1 in the textbook. Results are shown as line 4 in Table 5.3 below. The total absorption (air plus room surfaces) is calculated using equation (6.9) in the textbook and is shown in line 5 in Table 5.3 below. The octave band reverberant field sound pressure levels are calculated using Equations (6.12) and (1.63) in the textbook to give:

$$L_p = 10\log_{10}\langle p^2\rangle + 94 = 10\log_{10}\left[\frac{4W\rho c(1-\bar{\alpha})}{S\bar{\alpha}}\right] \text{ dB.}$$

Results for the reverberant field octave band $L_p$ are in line 7 of Table 5.3 below.

(b) The overall A-weighted level is calculated by adding logarithmically the octave band levels that have been corrected according to the A-weighting at the octave band centre frequency (see Equation (1.74) and Table 2.4 in the textbook). Thus:

$$L_{p,\text{tot}} = \log_{10}\left[\sum_{i=1}^{6} 10^{(L_{pi}+L_{Awt})/10}\right]$$

where the sum is over the 6 octave bands in Table 5.3 below. The A-weighted sound pressure level for each octave band is shown in Table 5.3, line 8 below.

(c) It can be seen that the A-weighted level is dominated by the levels in the 125 Hz, 250 Hz, 500 Hz and 1000 Hz octave bands. The required average Sabine absorption coefficient values to reduce the reverberant field sound pressure level are calculated for each octave band by:

$$\Delta L_p = 10\log_{10}\left[\frac{[(1-\bar{\alpha})/\bar{\alpha}]_{\text{old}}}{[(1-\bar{\alpha})/\bar{\alpha}]_{\text{new}}}\right] = 5. \quad \text{Thus, } [(1-\bar{\alpha})/\bar{\alpha}]_{\text{new}} = \frac{[(1-\bar{\alpha})/\bar{\alpha}]_{\text{old}}}{10^{5/10}}$$

which gives or the total required new space averaged absorption coefficient as:

$$\bar{\alpha}_{\text{new}} = \frac{\bar{\alpha}_{\text{old}} \times 10^{5/10}}{1 - \bar{\alpha}_{\text{old}}(1 - 10^{5/10})}$$

Assuming we would like a 5 dB reduction in each octave band, the required new value of the space averaged absorption coefficient, $\bar{\alpha}$, is as shown in line 9 of Table 5.3 below. Taking into account air absorption, the required value of $\bar{\alpha}$ for the room surfaces is as shown in line 10 of Table 5.3 below.

The required area of absorbing material to satisfy the required new surface absorption coefficients is given by:

$$S_m(\alpha_m + \alpha_{\text{air}}) + (S - S_m)\bar{\alpha}_{\text{old}} = S\bar{\alpha}_{\text{new}}$$

where $S$ is the existing room surface area and $S_m$ is the area of sound absorbing material of absorption coefficient, $\alpha_m$, to be used. The reduction in $S$ due to covering some part of the room surfaces with sound absorbing material is taken into account in writing the above expression, which can be rewritten as:

$$S_m = \frac{S(\bar{\alpha}_{\text{new}} - \bar{\alpha}_{\text{old}})}{(\alpha_m + \alpha_{\text{air}} - \bar{\alpha}_{\text{old}})},$$ with the results shown in line 11 of Table 5.3 below.

However, we need a single value of $S_m$ that will result in an overall A-weighted reduction of 5 dB. For any specified area of material, $S_m$, the reduction in sound pressure level in a particular octave band is determined first by calculating the new space average sound absorption coefficient (including air absorption), $\bar{\alpha}_{\text{new}}$. Thus:

$$\bar{\alpha}_{\text{new}} = \frac{S_m(\alpha_m + \alpha_{\text{air}}) + (S - S_m)\alpha_{\text{old}}}{S}$$

The noise reduction in each octave band due to the new average absorption coefficient is:

$$\Delta L_p = 10\log_{10}\left[\frac{[(1-\bar{\alpha})/\bar{\alpha}]_{\text{old}}}{[(1-\bar{\alpha})/\bar{\alpha}]_{\text{new}}}\right]$$

The new values of octave band $L_p$ are adjusted by the relevant A-weighting and then added logarithmically to give the total A-weighted sound pressure level. For an overall A-weighted sound pressure level reduction of 5 dBA, the target sound pressure level is

71.4 dBA. We can use trial and error (with excel) and the above equation to find the area of absorbing material required. The result is 72 $m^2$ and the new sound pressure level is shown in line 12 of Table 5.3 below, where it can be seen that a reduction of 5 dBA is achieved. The ceiling area is 80 $m^2$, so covering the entire ceiling would be appropriate. Alternatively, sound absorbing panels could be hung from the ceiling and the calculations repeated to determine the area needed, noting that in this case, none of the existing surfaces are covered with material.

**TABLE 5.3** Results for Problem 9

| | Octave band centre frequency (Hz) | | | | | | |
|---|---|---|---|---|---|---|---|
| | 125 | 250 | 500 | 1000 | 2000 | 4000 | Overall |
| Octave band sound power level (dB re $10^{-12}$ W) | 90.0 | 84.0 | 82.0 | 80.0 | 78.0 | 76.0 | |
| Octave band sound sound power (W) | 1.0E-03 | 2.51E-04 | 1.58E-04 | 1.0E-04 | 6.31E-05 | 3.98E-05 | |
| Existing $\alpha_{\text{surface}}$ | 0.08 | 0.1 | 0.15 | 0.2 | 0.3 | 0.4 | |
| Air attenuation coefficient, $m$ | 0.4 | 1.2 | 2.8 | 5.0 | 10.0 | 28.1 | |
| $\alpha_{\text{tot}}$ | 0.0808 | 0.102 | 0.155 | 0.210 | 0.319 | 0.454 | |
| Rockwool Sabine absorption coefficient | 0.1 | 0.6 | 1.0 | 1.0 | 1.0 | 1.0 | |
| Reverberant sound pressure level (dB re 20 µPa) | 84.9 | 77.8 | 73.7 | 70.1 | 65.7 | 61.2 | 86.1 |
| A-weighted Reverberant sound pressure level (dB re 20 µPa) | 68.8 | 69.2 | 70.5 | 70.1 | 66.9 | 62.2 | 76.4 |
| $[\bar{\alpha}_{\text{new}}]_{\text{tot}}$ | 0.217 | 0.265 | 0.368 | 0.456 | 0.597 | 0.725 | |
| $[\bar{\alpha}_{\text{new}}]_{\text{surface}}$ | 0.217 | 0.263 | 0.362 | 0.447 | 0.578 | 0.670 | |
| $S_m$ for 5 dB reduction ($m^2$) | 1038.8 | 49.4 | 38.0 | 46.8 | 60.4 | 68.5 | |
| New $L_p$ for 72 $m^2$ of material | 84.4 | 71.3 | 65.4 | 62.8 | 60.9 | 56.9 | 71.4 |

# 6

# Solutions to Additional Problems in Chapter 6

## Problem 1

(a) Using Equation (6.13), it can be seen that the direct and reverberant fields are equal when $\dfrac{Q}{4\pi r^2} = \dfrac{4(1-\bar{\alpha})}{S\bar{\alpha}}$

That is: $r = \sqrt{\dfrac{QS\bar{\alpha}}{16\pi(1-\bar{\alpha})}} = \sqrt{\dfrac{2 \times 300 \times 0.25}{16 \times \pi \times 0.75}} = 2.0$ m

(b) At 200 Hz, the wavelength is $c/f = 343/200 = 1.72$ m, $\lambda/2 = 0.86$ m and $\lambda/6 = 0.29$ m , so the microphone location is in the transition between near and far field and also in the direct field of the source (see above).

## Problem 2

(a) Existing $L_p = 76.0$ dBA. Allowed total $L_p = 85$ dBA. 5 new printers, so using the discussion in Section 1.12.3 in the textbook, the allowed $L_p$ from each new printer is:

$$L_p = 10\log_{10}\left(10^{8.5} - 10^{7.6}\right) - 10\log_{10}(5) = 77.4 \text{ (dBA re } 20\,\mu\text{Pa)}$$

(b) From Equation (3.60), the corresponding allowed sound power level of each printer is then:

$$L_W = 77.4 + 92.0 - 87.2 = 82.2 \text{ (dBA re } 10^{-12}\text{ W)}$$

## Problem 3

Room surface area $= 2(16 \times 16 + 2 \times 16 \times 6) = 896$ (m$^2$)

Using Equation (6.13), $L_p = L_W + 10\log_{10}\left(\dfrac{Q}{4\pi r^2} + \dfrac{4(1-\bar{\alpha})}{S\bar{\alpha}}\right) + 10\log_{10}\left(\dfrac{\rho c}{400}\right)$

Thus:

$$102 - 113 = 10\log_{10}\left[\left(\frac{2}{4\pi \times 2^2} + \frac{4(1-\bar{\alpha})}{896 \times \bar{\alpha}}\right) \times \left(\frac{413}{400}\right)\right]$$

and $10^{-11/10} - \dfrac{1}{8\pi} = \dfrac{4(1-\bar{\alpha})}{896\bar{\alpha}} \times \dfrac{413}{400}$

Thus, $4 - 4\bar{\alpha} = 896\bar{\alpha} \times 0.039645 \times \dfrac{400}{413}$

So $\bar{\alpha} = 0.11$

## Problem 4

**Near field:** this is the part of the sound field close to the sound source where the sound pressure and particle velocity are not in phase.
**Far field:** this is the part of the sound field at a long distance from the source where the sound pressure and particle velocity are in phase.
**Transition:** this is the part of the field where there is a gradual transition from near to far field.
**Direct:** this is the part of the sound field radiated directly from the source with no contribution from reflected waves.
**Reverberant:** this is the part of the sound field that consists of reflected waves only.

## Problem 5

(a) SPL due to 1 machine $= 90 - 10\log_{10} 15 = 90 - 11.76 = 78.24$ (dB re 20 µPa).
$10\log_{10} N = 85 - 78.24 = 6.76$
Thus, $N = 10^{6.76/10} = 4.74$
So need to remove 11 machines

(b) The required reduction in the reverberant field level is $90-85 = 5$ dB. Using Equation (6.13):
$\Delta = L_{p0} - L_{pf} = 10\log_{10}\left[\dfrac{R_f}{R_0}\right] = 10\log_{10}\left[\dfrac{1-\bar{\alpha}_0}{S\bar{\alpha}_0} \times \dfrac{S\bar{\alpha}_f}{1-\bar{\alpha}_f}\right]$
where the subscript 0 refers to the original configuration and the subscript $f$ refers to the final configuration. The room constant, $R$ is defined in equation (6.14). Thus, Rearranging the equation above gives:
$10^{5/10} = 3.162 = \dfrac{0.95}{0.05} \times \dfrac{\bar{\alpha}_f}{1-\bar{\alpha}_f}$
Rearranging further gives:
$\dfrac{\bar{\alpha}_f}{1-\bar{\alpha}_f} = \dfrac{3.162 \times 0.05}{0.95} = 0.1664$
so $(1 + 0.1664)\bar{\alpha}_f = 0.1664$ and $\bar{\alpha}_f = 0.14$.

## Problem 6

From Equation (6.13), $L_p = L_W + 10\log_{10}\left(\dfrac{Q}{4\pi r^2} + \dfrac{4(1-\bar{\alpha})}{S\bar{\alpha}}\right) + 10\log_{10}\left(\dfrac{413.6}{400}\right)$

$S = 2(10 \times 10 + 2 \times 10 \times 5) = 400$ m$^2$; $\quad Q = 2$

Thus, $85 = 95 + 10\log_{10}\left(\dfrac{2}{4\pi \times 2^2} + \dfrac{4(1-\bar{\alpha})}{400\bar{\alpha}}\right) + 0.15$

Thus, $\left(\dfrac{2}{4\pi \times 2^2} + \dfrac{4(1-\bar{\alpha})}{400\bar{\alpha}}\right) = 10^{(85-95-0.15)/10} = 0.09661$

Rearranging gives, $\dfrac{1-\bar{\alpha}}{100\bar{\alpha}} = 0.09661 - 0.03979,$

Therefore, $1 - \bar{\alpha} = 5.68\bar{\alpha}$,
so $\bar{\alpha} = \dfrac{1}{6.68} = 0.15$

## Problem 7

(a) Temperature is 600 °C; thus, from Equation (1.1), the speed of sound is:
$c = \sqrt{\gamma RT/M} = \sqrt{1.402 \times 8.314 \times 873/0.029} = 592$ (m/s)
Also from equation (1.1): $\gamma RT/M = \gamma P/\rho$

Thus: $\rho = \dfrac{PM}{RT} = \dfrac{101.4 \times 10^3 \times 0.029}{8.314 \times 873} = 0.405$ (kg/m$^3$)

(b) Acoustic power, $W = 10^6 \times 0.01 = 10,000$ W
The power is divided equally into the 8 octave bands so the power in the 500 Hz band is 10,000/8 (W).
From Equation (1.65), the sound power level, $L_W = 120 + 10\log_{10}(10,000/8) = 151.0$ (dB re $10^{-12}$ W)
The internal surface area of the chamber is $2(3 \times 3 + 6 \times 3 + 6 \times 3) = 90$ (m$^2$)
From Equation (6.13), the sound pressure level in the chamber is then:

$$L_p = L_W + 10\log_{10}\left[\frac{1}{4\pi r^2} + \frac{4(1-\bar{\alpha})}{S\bar{\alpha}}\right] + 10\log_{10}\left[\frac{\rho c}{400}\right]$$

$$= 151.0 + 10\log_{10}\left[\frac{1}{4\pi \times 2^2} + \frac{4(1-0.03)}{90 \times 0.03}\right] + 10\log_{10}\left[\frac{592 \times 0.405}{400}\right]$$

$$= 151.0 + 1.63 - 2.22 = 150.4 \text{ (dB re 20 µPa)}.$$

(c) To find the resonance instability frequency, we can treat the chamber as a tube closed at both ends. From Equation (1.40):

$f = \dfrac{c}{2L} = \dfrac{592.36}{2 \times 6} = 49.4$ (Hz)

Using Equation (1.68), we obtain:
$L_p = L_I - 26 + 10\log_{10}\rho c = 105.7 - 26 + 10\log_{10}(329) = 104.9$ (dB re 20 µPa).

## Problem 8

(a) Sound pressure due to machine only in each octave band is obtained by following the discussion in Section 1.12.3 in the textbook. Thus:
$L_{p,\text{machine only}} = 10\log_{10}\left[10^{L_{p,\text{on}}/10} - 10^{L_{p,\text{off}}/10}\right]$
The A-weighted sound pressure level in each octave band is found by adding the A-weighting correction in line 4 of Table 6.1 below. The overall A-weighted sound pressure level is found by adding the octave band values in line 4 of the table logarithmically according to Equation (1.74) in the textbook.

The result is 78.1 (dBA re 20 µPa).

(b) Using Equations (1.57) and (1.61), the sound power for hemispherical radiation is given by:
$W = 2\pi r^2 p_{\text{RMS}}^2/\rho c$

Taking logs of that equation, multiplying each resulting term by 10 and including

**TABLE 6.1** Data and results for Problem 8

| | Octave band centre frequency (Hz) | | |
|---|---|---|---|
| | 125 | 250 | 500 |
| Sound pressure level with machine on (dB re 20 µPa) | 91 | 84 | 80 |
| Sound pressure level with machine off (dB re 20 µPa) | 89 | 80 | 75 |
| Sound pressure level due to machine alone (dB re 20 µPa) | 86.7 | 81.8 | 78.3 |
| A−weighting (dB) | −16.1 | −8.6 | −3.2 |
| A−weighted level (dB re 20 µPa) | 70.6 | 73.2 | 75.1 |

the difference in reference levels between sound pressure and sound power, gives the following equation for the sound power level:

$$L_W = L_p + 20\log_{10} r + 10\log_{10} 2\pi + 10\log_{10}(400/\rho c)$$
$$= 78.1 + 20 + 8.0 = 106.1 \text{ (dBA re } 10^{-12}\text{ W)}$$

(c) Assumptions:

(i) $\rho c = 400$ kg m$^{-2}$ s$^{-1}$, which means ambient temperature = 20°C;

(ii) concrete ground results in 100% reflection;

(iii) other excess attenuation effects are small enough to be ignored; and

(iv) no reflecting surfaces nearby, other than the ground.

(d) Using Equation (1.75):

$$\begin{aligned}\text{NR} &= 10\log_{10}[10 - 0/10 + 10 - 0/10] \\ &\quad - 10\log_{10}[10 - 6/10 + 10 - 10/10 \\ &\quad + 10 - 12/10 + 10 - 14/10 + 10 - 15/10 + 10 - 16/10] \\ &= 3.0 + 2.9 = 5.9 \text{ dB noise reduction}\end{aligned}$$

## Problem 9

(a) From Equation (1.65), the sound power level is:

$$L_W = 10\log_{10} W + 120 = 10\log_{10} 3.1 + 120 = 124.9 \text{ (dB re } 10^{-12}\text{ W)}$$

From Equation (1.1), $c = \sqrt{\dfrac{\gamma RT}{M}} = \sqrt{\dfrac{1.4 \times 8.314 \times 1473}{0.035}} = 700$ (m/s)

(b) Rearranging Equation (1.1) gives $\rho = \dfrac{\gamma P}{c^2} = \dfrac{1.4 \times 101.4 \times 10^3}{700^2} = 0.29$ (kg/m$^3$)

Surface area, $S = \pi dL + \pi d^2/2 = \pi \times 4 \times 6 + \pi \times 16/2 = 32\pi = 100$ (m$^2$)

From Equation (6.13), with the direct field term omitted:

$$L_p = L_W + 10\log_{10}\left[\frac{4(1-\bar{\alpha})}{S\bar{\alpha}}\right] + 10\log_{10}\left[\frac{\rho c}{400}\right]$$
$$= 124.9 + 10\log_{10}\left[\frac{4 \times 0.95}{100 \times 0.05}\right] + 10\log_{10}\left[\frac{700 \times 0.29}{400}\right] = 120.8 \text{ (dB re 20 µPa)}$$

## Problem 10

(a) From Table 2.21 in the textbook, the allowable community noise level to ensure minimal risk of complaints is $40 + 15 - 10 = 45$ (dBA re 20 µPa). If the only noise is in the 500 Hz octave band then the $-3.2$ dBA-weighting at this frequency, results in an allowable level of 48.2 (dB re 20 µPa) in that band. The existing level is 44 dB so the allowed increase is 4.2 dB. A reverberant field sound pressure level corresponds to a sound power level (see Equation (6.13) in the textbook) of:

$$L_W = 88 - 10\log_{10}\left(\frac{4(1-\bar{\alpha})}{S\bar{\alpha}}\right) - 0.15 \text{ (dB re } 10^{-12}\text{ W)}$$

Room 25 m $\times$ 20 m $\times$ 8 m,
$S = 2(25 \times 20 + 25 \times 8 + 20 \times 8) = 1720$ (m$^2$).
$V = 25 \times 20 \times 8 = 4000$ (m$^3$)
$T_{60} = 2.1$ secs. From Equation (6.15) in the text:

$$\bar{\alpha} = \frac{55.25 \times 4000}{343 \times 2.1 \times 1720} = 0.178$$

Thus: $L_W = 88 - 10\log_{10}\left(\frac{4(1-0.178)}{1720 \times 0.178}\right) - 0.15 = 107.5$ (dB re $10^{-12}$ W)

This is the sound power level of the existing equipment. The allowable total sound power of existing + new equipment is $107.5 + 4.2 = 111.9$ (dB re $10^{-12}$ W). From the discussion in Section 1.12.3 in the textbook, the allowed power level for the new equipment is:

$$L_W = 10\log_{10}\left(10^{111.9/10} - 10^{107.9/10}\right) = 109.7 \text{ (dB re } 10^{-12}\text{ W)}$$

As there are 5 new machines, the allowed power level for each is:

$$L_W = 109.7 - 10\log_{10}(5) = 102.7 \text{ (dB re } 10^{-12}\text{ W)}.$$

**Assumptions**

(i) Only absorption is due to floor, walls and ceiling.

(ii) Air temperature of 20° C.

(iii) The relative contribution of direct and reverberant sound energy to the community noise levels will be the same for the new machines as for the old machines.

(iv) Sabine type room

(b) If ceiling tile with $\bar{\alpha} = 0.5$ were added, then the new $S\bar{\alpha}$ is:
$S\bar{\alpha} = 500 \times 0.5 + (1720 - 500) \times 0.178 = 467.2$ (m$^2$), corresponding to
$\bar{\alpha} = 467.2/1720 = 0.272$.
From equation (6.14), the old room constant, $R = (1720 \times 0.178)/(1 - 0.178) = 372.5$ (m$^2$) and the new $R = (1720 \times 0.272)/(1 - 0.272) = 642.6$ (m$^2$).

Thus the allowed increase in sound power level for the same reverberant field sound pressure level is:

$$\Delta L_W = 10\log_{10}(642.6) - 10\log_{10}(372.5) = 2.4 \text{ (dB re } 10^{-12}\text{ W)}$$

Assuming that the reverberant field dominates the direct field, the new allowed power level of each machine is $102.7 + 2.4 = 105.1$ (dB re $10^{-12}$ W).

## Problem 11

(a) $r = 2.5$ m, $\ell = 0.5$ m and from Equation (3.42), the criteria for far field are:
$r \gg \lambda/(2\pi), \quad r \gg \ell, \quad r \gg \pi\ell^2/(2\lambda)$
At 500 Hz, $\lambda = 343/500 = 0.686$ m and $\lambda/2\pi = 0.109$
$\dfrac{\pi\ell^2}{2\lambda} = \dfrac{\pi \times 0.5^2}{2 \times 0.686} = 0.57$
If we use a factor of 3 to define "much greater", then it is clear that the above three conditions are satisfied and the measurement location is in the far field.

(b) $T_{60} = 2.5$ seconds
$V = 480$ m$^3$
$S = 2 \times (10 \times 8 + 6 \times 8 + 10 \times 6) = 376$ (m$^2$)
From Equation (6.15), $S\bar{\alpha} = \dfrac{55.25V}{cT_{60}} = \dfrac{55.25 \times 480}{343 \times 2.5} = 30.927$
Thus, $\bar{\alpha} = 30.927/376 = 0.0822$
From Equation (1.65), $L_W = 120 + 10\log_{10} 1 = 120$ (dB re $10^{-12}$ W)
From Equation (6.13), $L_p = L_W + 10\log_{10}\left(\dfrac{D_\theta}{4\pi R^2} + \dfrac{4(1-\bar{\alpha})}{S\bar{\alpha}}\right) + 10\log_{10}\left(\dfrac{\rho c}{400}\right)$

$$\text{Thus, } L_p = 120 + 10\log_{10}\left(\frac{1}{4\pi \times 2.5^2} + \frac{4(1-0.0822)}{30.927}\right) + 10\log_{10}\left(\frac{413.6}{400}\right)$$
$$= 111.3 \text{ (dB re } 20\,\mu\text{Pa)}$$

(c) The sound intensity due to the reverberant field = 0
From Equation (1.61), the sound intensity, $I$, due to the direct field is:
$I = \dfrac{W}{4\pi r^2} = \dfrac{1}{4\pi \times 2.5^2} = 0.01273$ (W/m$^2$)
From Equation (1.67), the intensity level is:
$L_I = 10\log_{10} I + 120 = 101.0$ (dB re $10^{-12}$ W/m$^2$)
Assuming plane waves, the energy density from Equation (1.47) is $\psi = \dfrac{\langle p^2 \rangle}{\rho c^2}$
where the sound pressure squared is obtained from Equation (1.62) as:
$\langle p^2 \rangle = p_{ref}^2 \times 10^{L_p/10} = 4 \times 10^{-10} \times 10^{111.3/10} = 53.96$ (Pa$^2$)
Thus $\psi = \dfrac{53.96}{1.206 \times 343^2} = 3.80 \times 10^{-4}$ (J/m$^3$)

(d) Existing $\bar{\alpha} = 0.0822$
Floor area $= 10 \times 8 = 80$ (m$^2$)
From Equation (5.66), the new $\bar{\alpha} = \dfrac{0.9 \times 80 + 0.0822(376 - 80)}{376} = 0.256$

$$\text{From Equation (6.13), the new } L_p = 120 + 10\log_{10}\left(\frac{1}{4\pi \times 2.5^2} + \frac{4(1-0.256)}{0.256 \times 376}\right) + 0.1$$
$$= 120 + 10\log_{10}(0.0127 + 0.03092) = 106.5 \text{ (dB re } 20\,\mu\text{Pa)}$$

So covering the floor with sound absorbing material would decrease the sound pressure level in the room by 4.8 dB

## Problem 12

(a) As can be seen from inspection of Table 1.2 in the textbook, the 125 Hz octave band spans 88 Hz to 176 Hz
Area of room $= 2(5 \times 5 + 5 \times 3 \times 2) = 110$ (m$^2$)
Perimeter of room $= 4(5 + 5 + 3) = 52$ m
Volume of room $= 5 \times 5 \times 3 = 75$ (m$^3$)

Number of modes below 88 Hz is given by Equation (6.3) in the textbook as:

$$N = \frac{4\pi \times 88^3 \times 75}{3 \times 343^3} + \frac{\pi \times 88^2 \times 110}{4 \times 343^2} + \frac{88 \times 52}{8 \times 343} = 5.3 + 5.7 + 1.7 = 12.7$$

Number of modes below 176 Hz is given by Equation (6.3) in the textbook as:

$$N = \frac{4\pi \times 176^3 \times 75}{3 \times 343^3} + \frac{\pi \times 176^2 \times 110}{4 \times 343^2} + \frac{176 \times 52}{8 \times 343} = 42.4 + 22.7 + 3.3 = 68.4$$

So the number of modes in the 125 Hz octave band is 55 or 56.

(b) Absorption coefficient = 0.15
From equation (6.15), the reverberation time is:

$T_{60} = 55.25 \times 75/(110 \times 343 \times 0.15) = 0.732$ (s)

From Equation (6.4), the modal density at 125 Hz is:

$$\frac{\mathrm{d}N}{\mathrm{d}f} = \frac{4\pi \times 125^2 \times 75}{343^3} + \frac{\pi \times 125 \times 110}{2 \times 343^2} + \frac{52}{8 \times 343} = 0.365 + 0.184 + 0.019 = 0.568$$

Thus, from equation (6.7), the modal overlap, $M = \dfrac{2.2}{0.732} \times 0.568 = 1.707$

(c) The number of modes in the band exceeds 6 so no problems with octave band sound power measurements.

(d) The modal overlap is 1.7 which is less than 3 (textbook, page 262) and thus too small for the room to be used for tonal sound power measurements at 125 Hz.

## Problem 13

(a) Room volume $V = 30$ m$^3$, surface area, $S = 50$ m$^2$, source sound power = 1 W.
Using Equation (1.62), the reverberant field mean square pressure is:

$\langle p^2 \rangle = 4 \times 10^{-10} \times 10^{116/10} = 159.2$ Pa$^2$

Using Equation (6.12) in the textbook we obtain:

$$\frac{1 - \bar{\alpha}}{\bar{\alpha}} = \frac{159.2 \times 50}{4 \times 1.206 \times 343 \times 1} = 4.812$$

Thus $\bar{\alpha} = 0.172$

(b) Using Equation (6.15) in the textbook:

$$T_{60} = \frac{55.25 \times 30}{343 \times 50 \times 0.172} = 0.56 \text{ s}$$

Thus, $T_{10} = 0.56/6 = 0.094$ s.

(c) Using Equation (5.66), the new $\bar{\alpha}$ is given by:

$$\bar{\alpha} = \frac{10 \times 0.8 + 40 \times 0.172}{50} = 0.298$$

Using Equation (6.13) in the textbook, we obtain for the new mean square reverberant sound pressure:

$$\langle p^2 \rangle = \frac{4 \times 1 \times 1.206 \times 343 \times (1 - 0.298)}{50 \times 0.298} = 77.96 \text{ Pa}^2$$

From Equation (1.63), the corresponding sound pressure level is then:

$L_p = 94 + 10\log_{10}(77.96) = 112.9$ (dB re 20 µPa)

which corresponds to a reduction of $116 - 112.9 = 3.1$ dB.

## Problem 14

Assume that the house is approximately in a direction along the normal axis from the window. The power incident on the window is the intensity in the direction of the window multiplied by the area of the window. Thus, using Equations (1.61) and (6.8) in the textbook:

$$W = \frac{\langle p_i^2\rangle S}{4\rho c}$$

where $S$ is the area through which the sound is being radiated.
From Equation (7.11), the power radiated through the window is:

$$W = \frac{\tau\langle p_i^2\rangle S}{4\rho c}$$

where $\tau = 10^{-TL/10} = 10^{-2.7} = 1.995 \times 10^{-3}$.

The reverberant sound pressure level is 88 (dB re 20 µPa).

Thus $\langle p_i^2\rangle = 4 \times 10^{-10} \times 10^{88/10} = 0.252$ (Pa$^2$)

The power radiated through the window is then:

$$W = \frac{1.995 \times 10^{-3} \times 0.252 \times 1.5}{4 \times 1.206 \times 343} = 0.457\,(\mu\mathrm{W})$$

For an incoherent plane source, the on-axis sound pressure at the receiver is given by Equation (3.38) in the textbook. The quantity, $r/\sqrt{HL} = 60/1.5 = 40$. Thus from figure 3.18 in the textbook, it is clear that we can treat it as a hemispherically radiating point source producing a sound pressure described by equation (3.38) in the textbook. Thus:

$$\langle p^2\rangle = \frac{1.206 \times 343 \times 4.565 \times 10^{-7}}{2 \times \pi \times 60^2} = 8.349 \times 10^{-9}\ (\mathrm{Pa}^2)$$

From Equation (1.63), this corresponds to a sound pressure level of:

$L_p = 10\log_{10}(8.349 \times 10^{-9}) + 94 = 13.2$ (dB re 20 µPa)

As the ground is hard asphalt, we may add 3 dB to the level to account for the effect of ground reflection (see Section 4.5.1 in the textbook). Thus the expected level at the house is 16 dB.

## Problem 15

Sound intensity in any one direction in a reverberant field is given by:

$I = \psi c/4 = 1.5 \times 10^{-3} \times 343/4 = 0.1286$ (W/m$^2$)

Sound power is: $W = IS = 0.1286 \times 1.5 = 0.1929$ (W)

Sound power level is: $L_W = 120 + 10\log_{10}(0.1929) = 112.9$ (dB re $10^{-12}$ W)

**Assumptions:**

(a) temperature of 20°C, so $c = 343$ m/s;
(b) enclosure is air filled.

# Problem 16

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(a) This is a flat room as described in Section 6.5 in the textbook. The energy reflection coefficient, $\beta = 0.7^2$ and $\bar{\alpha} = 1 - 0.7^2 = 0.51$
Room height, $a = 5$ m and distance $r = 5$ m. Thus $r/a = 1$. From Figure 6.8 in the textbook, the reverberant field sound pressure is given by:

$$10\log_{10}\langle p^2\rangle = 10\log_{10}\left[\frac{W\rho c}{\pi a^2}\right] - 8$$

Taking logs, multiplying each term by 10 and accounting for the differences in sound pressure and sound power reference quantities, the reverberant field sound pressure level is:

$$\begin{aligned} L_{p(\text{reverb})} &= L_W + 10\log_{10}\left(\frac{\rho c}{400}\right) - 10\log_{10}(\pi a^2) - 8 \\ &= 130 + 0.14 - 18.95 - 8 = 103.2 \text{ (dB re } 20\,\mu\text{Pa)} \end{aligned}$$

The direct field sound pressure level is obtained using equations (1.57) and (1.61), and performing the same operations as used to obtain the above equation. Thus:

$$\begin{aligned} L_{p(\text{direct})} &= L_W + 10\log_{10}\left(\frac{\rho c}{400}\right) - 10\log_{10}(4\pi r^2) \\ &= 130 + 0.14 - 24.97 = 105.2 \text{ (dB re } 20\,\mu\text{Pa)} \end{aligned}$$

(b) Sabine room
Area, $S = 2[10 \times 10 + 10 \times 5 \times 2] = 400$ (m$^2$)
and volume, $V = 10 \times 10 \times 5 = 500$ m$^2$
The total sound pressure level (direct plus reverberant) in the room is given by equation (6.13) as:

$$\begin{aligned} L_p &= L_W + 10\log_{10}\left[\frac{D}{4\pi r^2} + \frac{4(1-\bar{\alpha})}{S\bar{\alpha}}\right] \\ &= 130 + 10\log_{10}\left[\frac{1}{4\pi \times 25} + \frac{4 \times 0.49}{400 \times 0.51}\right] = 111 \text{ (dB re } 20\,\mu\text{Pa)} \end{aligned}$$

(c) From Equation (6.13), the direct and reverberant fields equal when:
$D/4\pi r^2 = 4(1-\bar{\alpha})/S\bar{\alpha}$ . Thus:

$$r = \sqrt{\frac{DS\bar{\alpha}}{4\pi \times 4(1-\bar{\alpha})}} = \sqrt{\frac{400 \times 0.51}{16\pi \times 0.49}} = 2.9 \text{ m}$$

(d) From Equation (6.15), $T_{60} = \dfrac{55.25V}{Sc\bar{\alpha}} = \dfrac{55.25 \times 500}{400 \times 343 \times 0.51} = 0.39$ s

(e) Treat the room like an enclosure. Thus, the sound level at the receiver without the enclosure at a distance of 50+5 m is found using Equations (1.57) and (1.61), adjusting Equation (1.61) for hemispherical radiation (multiplying the RHS by 2) and then converting to levels to give:

$$L_p = L_W + 10\log_{10}(\rho c/400) - 10\log_{10} 2\pi r^2 = 130 + 0.14 - 42.8 = 87.4 \text{ (dB re } 20\,\mu\text{Pa)}$$

The enclosure noise reduction is given by Equation (7.29) in Chapter 7 in the textbook and is NR = TL - $C$, where $C$ is given by Equation (7.30) as:

$$C = 10\log_{10}\left[0.3 + \frac{S_E(1-\bar{\alpha})}{S_i\bar{\alpha}}\right] = 10\log_{10}\left[0.3 + \frac{300 \times 0.49}{400 \times 0.51}\right] = 0.1 \text{ dB}$$

Thus the noise reduction = 24.9 dB and the sound pressure level at 50 m is:

$L_p = 87.4 - 24.9 = 62.5$ (dB re 20 µPa).

## Problem 17

(a) Room 8 m × 6 m × 3 m, $S = 2(8 \times 6 + 8 \times 3 + 6 \times 3) = 180$ (m$^2$).

The mean Sabine absorption coefficient may be calculated using Equation (5.66) in the text to give:

$$\bar{\alpha} = \frac{48 \times 0.15 + 132 \times 0.05}{180} = 0.0767$$

Using equation (6.12), we can write:

$$\langle p^2 \rangle_R = \frac{4 \times 25 \times 10^{-3} \times 1.206 \times 343(1 - 0.0767)}{180 \times 0.0767} = 2.766 \text{ (Pa}^2\text{)}$$

The reverberant field sound pressure level is then:

$$L_{pR} = 10 \log_{10}(2.766) + 94 = 98.4 \text{ (dB re } 20\,\mu\text{Pa)}$$

(b) Direct and reverberant fields equal (see Equation (6.13) in the text) when: $D_\theta/4\pi r^2 = 4/R$. Thus the distance, $r$, at which this occurs is:

$$r = \left(\frac{S\bar{\alpha}}{16\pi(1 - \bar{\alpha})}\right)^{1/2} = \left(\frac{180 \times 0.0767}{16\pi \times (1 - 0.0767)}\right)^{1/2} = 0.55 \text{ m}$$

## Problem 18

(a) Using Equation (3.51) in the textbook and substituting in the appropriate values gives:

$$L_W = 95 + 22.5 - 8.3 + 0.2 - 13.9 = 95.5 \text{ (dB re } 10^{-12}\text{ W)}$$

Using Equation (1.64), the radiated sound power in mW is:

$$W = 1000 \times 10^{95.5/10} \times 10^{-12} = 3.55 \text{ mW.}$$

(b) The room surface area is $S = 2(6.84 \times 5.565 + 6.84 \times 4.72 + 5.565 \times 4.72) = 193.2$ m$^2$.

Using equation (6.13) in the textbook, (and setting $\rho c = 413.6$) we can write:

$$L_p = 95.5 + 10 \log_{10}\left(\frac{2}{4\pi \times 0.5^2} + \frac{4(1 - 0.022)}{193.2 \times 0.022}\right) + 10 \log_{10}(\rho c/400)$$
$$= 97.6 \text{ (dB re } 20\,\mu\text{Pa)}$$

## Problem 19

(a) Considering only the reverberant field of the machine we may use Equation (6.12). That is, $\langle p^2 \rangle_R = 4W\rho c(1 - \bar{\alpha})/(S\bar{\alpha})$.
The surface area of the factory is $S = 2(10 \times 10 + 2 \times 10 \times 3) = 320$ m$^2$.
Using Equation (1.62), the sound pressure level, $L_p = 83$ dB re 20 µPa, corresponds to $\langle p^2 \rangle = 4 \times 10^{-10} \times 10^{83/10} = 7.98 \times 10^{-2}$ Pa$^2$.
Using Equation (6.12), the required mean absorption coefficient is given by:

$$\frac{\bar{\alpha}}{(1 - \bar{\alpha})} = \frac{4 \times 0.01 \times 1.206 \times 343}{7.981 \times 10^{-2} \times 320} = 0.648.$$

Thus, $\bar{\alpha} = 0.393$. This would be the required absorption coefficient if the floor were lined as well. If we assume that the concrete floor has an absorption coefficient of 0.01, and we let the required absorption coefficient of the walls and ceiling be $x$, then we can use Equation (5.66) to write $320 \times 0.393 = 100 \times 0.01 + 220x$, which gives $x = 0.567$. Thus the required absorption coefficient for the walls and ceiling is 0.57.

(b) From Equation (6.14), the room constant, $R = S\bar{\alpha}/(1 - \bar{\alpha}) = 320 \times 0.648 = 207$ m$^2$.
From Equation (1.65), the sound power level is given by:
$L_W = 10\log_{10}(0.01) + 120 = 100$ dB re $10^{-12}$ W.
For a total $L_p$ of 90 dB re 20 µPa we may use Equation (6.13) to write (assuming a directivity factor, $D_\theta = 2$, as the source is assumed close to a hard floor and other surfaces are more absorptive):

$$90 = 100 + 10\log_{10}\left(\frac{2}{4\pi r^2} + \frac{4}{207}\right).$$

Solving the above gives $r = 1.40$ m as the radius around the machine within which the sound pressure level will exceed 90 dB re 20 µPa.

# 7

# Solutions to Additional Problems in Chapter 7

## Problem 1

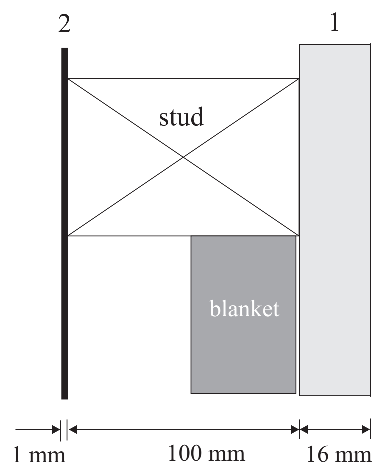


**FIGURE 7.1** Arrangement for Problem 1.

(a) Using Equation (7.4) and the data in Appendix A:

$$f_c(\text{steel}) = \frac{0.55 \times 343^2}{0.001 \times 5130/\sqrt{1-0.3^2}} = 12 \text{ (kHz)}$$

$$f_c(\text{gypsum}) = \frac{0.55 \times 343^2}{0.016 \times 1670/\sqrt{1-0.24^2}} = 2.35 \text{ (kHz)}$$

From Equation (7.25), $f_\ell = 55/0.1 = 550$ (Hz)

$m_{\text{steel}} = 7850 \times 0.001 = 7.85$ kg/m$^2$ and $m_{\text{gypsum}} = 760 \times 0.016 = 12.16$ (kg/m$^2$)

From Equation (7.24), $f_0 = 80\sqrt{\dfrac{12.16 + 7.85}{0.1 \times 12.16 \times 7.85}} = 115.8$ (Hz)

From the equations in the caption of Figure 7.13:

$TL_A = 20\log_{10}(7.85 + 12.16) + 20\log_{10} 115.8 - 48 = 26.02 + 41.27 - 48 = 19.3$ (dB)

From the equations in the caption of Figure 7.13:

$$TL_{B2} = 20\log_{10} 12.16 + 10\log_{10} 0.6 + 20\log_{10}(2351) + 10\log_{10}(12032) + 20\log_{10}\left[1 + \frac{7.85 \times 2351^{1/2}}{12.16 \times 12032^{1/2}}\right] - 78 \text{ (dB)}$$
$$= 21.7 - 2.2 + 67.4 + 40.8 + 2.2 - 78 = 51.9 \text{ dB at } 2351/2 = 1175 \text{ (Hz)}$$

From the equations in the caption of Figure 7.13:

$$TL_C = 51.9 + 6 + 10\log_{10} 0.02 + 20\log_{10}\left[\frac{12032}{2351}\right] = 55.1\text{(dB)} \quad \text{at } f = 12 \text{ (kHz)}$$

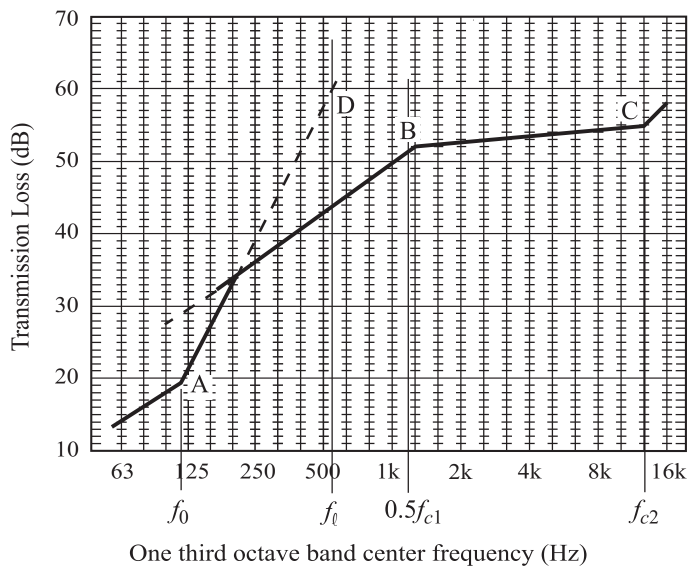


**FIGURE 7.2** Results for Problem 1.

**TABLE 7.1** Results from Figure 7.2 for Problem 1

| 1/3 octave centre frequency (Hz) | TL | 1/3 octave centre frequency (Hz) | TL |
|---|---|---|---|
| 400 | 40.5 | 1600 | 52.2 |
| 500 | 43 | 2000 | 52.8 |
| 630 | 45 | 2500 | 53 |
| 800 | 47.5 | 3150 | 53.2 |
| 1000 | 49.5 | 4000 | 53.5 |
| 1250 | 52 | | |

(b) $TL_{500} = -10\log_{10}\frac{1}{3}\left[10^{-40.5/10} + 10^{-43/10} + 10^{-45/10}\right] = 42.4$ (dB)

(c) $\tau_{\text{overall}} = \dfrac{10^{-20/10} \times 2 + 10^{-42.4/10} \times 8}{10} = 2.046 \times 10^{-3}$

So overall TL $= -10\log_{10}\tau = 27$ (dB)

# Problem 2

The enclosure TL = 30 dB in the 500 Hz 1/3 octave band. Thus the IL of the inlet and outlet mufflers should also be 30 dB. As the mufflers are dissipative and the TL is greater than 5 dB, the Insertion Loss will be the same as the TL; ie 30 dB.

Assuming 98% efficiency, power converted to heat = 1.6 kW

Rearranging Equation (7.37), we obtain for the required air flow rate:

$$V = \frac{1.6 \times 10^3}{5 \times 1010 \times 1.206} = 0.263 \text{ (m}^3\text{/s)}$$

The fan should be installed on the end of the duct that is closest to the inside of the enclosure and preferably on the outlet duct as the sucking action of the fan is likely to result in a more uniform air flow over the equipment in the enclosure. However, having a fan on the inlet duct (on the end inside the enclosure) is also a valid solution.

## Problem 3

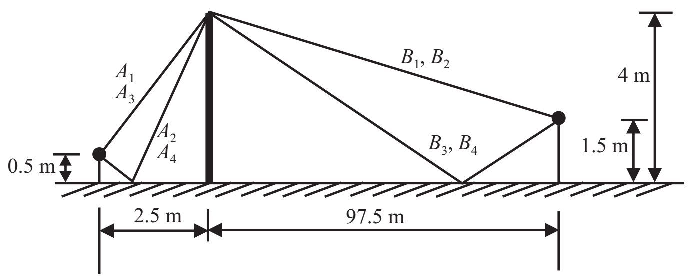


**FIGURE 7.3** Arrangement for Problem 3.

**Direct path over the top of the barrier.**

$A_1 = \sqrt{2.5^2 + 3.5^2} = 4.3012$ m

$B_1 = \sqrt{2.5^2 + 97.5^2} = 97.5321$ m

Source-receiver distance, $d_{\text{SR1}} = \sqrt{100^2 + 1^2} = 100.005$ (m)

**Reflected paths**

$A_2 = A_4 = \sqrt{2.5^2 + 4.5^2} = 5.1478$ m

$B_3 = B_4 = \sqrt{97.5^2 + 5.5^2} = 97.6550$ m

*Path involving reflection on the freeway side and not on the receiver side.*

$d_{\text{SR2}} = \sqrt{100^2 + 2^2} = 100.02$ m

*Path involving reflection on the receiver side, but not on the freeway side.*

$d_{\text{SR3}} = \sqrt{100^2 + 2^2} = 100.02$ m

*Path involving a reflection on both sides.*

$d_{\text{SR4}} = d_{\text{SR1}} = 100.005$ m

Reflection Loss on receiver side = 3 dB

At 500 Hz, $\lambda = 343/500 = 0.686$ m

**Direct path over the top:**

From Equation (7.38) and Figure 7.19:

$$N_1 = \frac{2}{\lambda}(A_1 + B_1 - d_{\mathrm{SR1}}) = \frac{2}{0.686}(4.301 + 97.532 - 100.005) = 5.3; \quad A_{b1} = \Delta_{b1} = 15.2 \text{ dB}$$

**Path with reflection on the freeway side:**

$$N_2 = \frac{2}{0.686}(5.148 + 97.532 - 100.02) = 7.8; \quad A_{b2} = \Delta_{b2} = 16.8 \text{ dB}$$

**Path with reflection on the receiver side:**

$$N_3 = \frac{2}{0.686}(97.655 + 4.301 - 100.02) = 5.6; \quad A_{b3} = \Delta_{b3} = 15.1 \text{ dB}$$

**Path with reflection on both sides:**

$$N_4 = \frac{2}{0.686}(5.148 + 97.655 - 100.005) = 8.2; \quad A_{b4} = \Delta_{b4} = 17.0 \text{ dB}$$

**Noise reductions:**

w/o barrier, 2 paths: 0 dB and 1.5 dB

with barrier, 4 paths: 15.2, 16.8, $(15.1 + 3)$, $(17 + 3)$

Overall NR=$A_b$, see Equation (7.51) in the textbook:

$$\begin{aligned} A_b &= 10\log_{10}\left(10^{-0/10} + 10^{-1.5/10}\right) - 10\log_{10}\left(10^{-15.2/10} + 10^{-16.8/10} + 10^{-18.1/10} + 10^{-20.0/10}\right) \\ &= 2.32 - (-11.16) = 13.5 \text{ (dB)} \end{aligned}$$

## Problem 4

From Equation (7.14), $\mathrm{TL} = \mathrm{NR} + 10\log_{10}\left(\frac{A}{S\bar{\alpha}}\right)$

$V = 72 \text{ m}^3$

$T_{60} = 2.1 \text{ s}$

From Equation (6.15), $S\bar{\alpha} = \frac{55.25V}{cT_{60}} = \frac{55.25 \times 72}{343 \times 2.1} = 5.523 \text{ (m}^2\text{)}$

Thus, $\mathrm{TL} = 31 + 10\log_{10}\left(\frac{10}{5.523}\right) = 33.6 \text{ (dB)}$

## Problem 5

(a) The normal incidence $\mathrm{TL}_N = 20$ dB. However as incident field is diffuse we need to use the field incidence TL which is 5.5 dB less. Thus:
Transmitted power level = incident power level − TL
$= 112.9 - (20 - 5.5) = 98.4 \text{ (dB re } 10^{-12}\text{ W)}$

(b) From Equation (1.64), the radiated sound power is:
$W = 10^{-12} \times 10^{98.4/10} = 6.92$ mW.
As the ground is hard, we need to include the reflected wave. However, the ground is further away than the source/receiver separation so we need to add the reflected wave separately.

Assume the following:

(i) The enclosure wall is large enough to be treated as an infinite baffle (at least 3 wavelengths in size);

(ii) For a temperature of 20°C, $\rho c = 413.6 \text{ (kg m}^{-2}\text{ s}^{-1}\text{)}$;

(iii) The sound transmitted through the enclosure walls is negligible;

At 1 m from the wall, on axis, according to Equation (3.38), the direct wave produces:

$$\langle p^2 \rangle = \frac{2\rho c W}{\pi H L} \tan^{-1}\left[\frac{HL}{2r\sqrt{H^2+L^2+4r^2}}\right]$$
$$= \frac{2 \times 413.6 \times 6.92 \times 10^{-3}}{\pi \times 1.5} \tan^{-1}\left[\frac{1.5}{2 \times 1\sqrt{1^2+1.5^2+4\times 1^2}}\right]$$
$$= 0.3299 \text{ Pa}^2$$

The distance travelled by the reflected wave is $2\sqrt{2^2+0.5^2} = 4.12$ m. At this distance, we can use the point source approximation. Thus, from Equation (3.39):

$$\langle p^2 \rangle = \frac{\rho c W}{2\pi r^2} = \frac{413.6 \times 6.92 \times 10^{-3}}{2\pi \times 4.12^2} = 2.684 \times 10^{-2} \text{ Pa}^2$$

Thus, the total pressure squared is 0.3567 (Pa$^2$).

From Equation (1.63), the sound pressure level is then:

$L_p = 94 + 10\log_{10}\langle p^2\rangle = 94.0 + 10\log_{10}(0.3567) = 89.5$ (dB re 20 µPa)

## Problem 6

(a) .

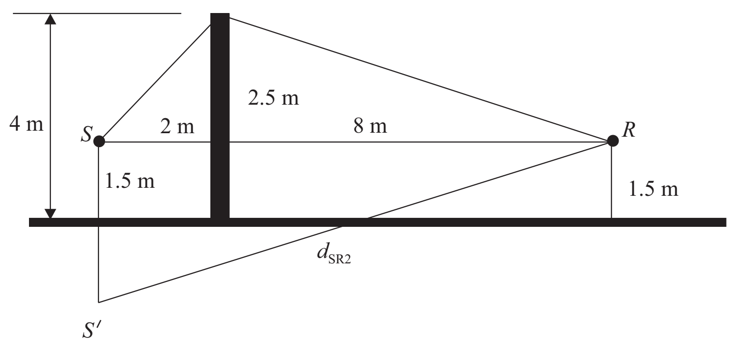


**FIGURE 7.4** Arrangement for Problem 6.

Assume temperature of 20°C and assume that the directivity is unchanged by the barrier. We also assume that the source is sufficiently small to be treated as a point source.

**Direct path over the top**

$d_{SR1} = 8 + 2 = 10$ m

$A_1 = \sqrt{2.5^2 + 2^2} = 3.2016$ m; $\quad B_1 = \sqrt{2.5^2 + 8^2} = 8.3815$ m

From Equation (7.38), $N_1 = \dfrac{2}{\lambda}(A_1 + B_1 - d_{SR1}) = \dfrac{2 \times 500}{343} \times 1.5831 = 4.6.$

Thus, from Figure 7.19, $\Delta_{b1} = 19.5$.

From Equation (7.39), $A_{b1} = 19.5 + 20\log_{10}(1.1583) = 19.5 + 1.3 = 20.8$ dB

**Reflected path**

$d_{SR2} = \sqrt{9 + 100} = 10.4403$ m

$A_2 = \sqrt{5.5^2 + 2^2} = 5.8523$ m; $\quad B_2 = B_1 = \sqrt{2.5^2 + 8^2} = 8.3815$ m

$N_2 = \frac{2}{\lambda}(A_2 + B_2 - d_{\mathrm{SR}2}) = \frac{2 \times 500}{343} \times 3.7935 = 11.0$

Thus, $\Delta_{b2} = 23$ dB and $A_{b2} = 23.0 + 20\log_{10}(1.3633) = 23.0 + 2.7 = 25.7$ dB

From Equation (7.51):

$$A_b = 10\log_{10}\left(10^{0/10} + 10^{0/10}\right) - 10\log_{10}\left(10^{-20.8/10} + 10^{-25.7/10}\right)$$
$$= 3.0 + 19.6 = 22.6 \text{ dB}$$

(b) If traffic is the noise source, then we must use the dashed curve in Figure 7.19. Thus:

$\Delta_{b1} = 14.5$ dB and $\Delta_{b2} = 18$ dB.

Following the procedure above to calculate $A_{b1}$ and $A_{b2}$, we obtain:

$A_{b1} = 14.5 + 1.3 = 15.8$ dB and $A_{b2} = 18 + 2.7 = 20.7$ dB.

Thus the barrier noise reduction for a traffic noise source is obtained from Equation (7.51) as:

$A_b = 3 - 10\log_{10}\left[10^{-15.8/10} + 10^{-20.7/10}\right] = 3.0 + 14.6 = 17.6$ dB.

## Problem 7

Assume that sound propagation around the ends of the barrier is negligible and that air absorption is the same with and without the barrier.

First calculate the loss due to ground reflection in the absence of the barrier - then calculate the barrier attenuations for each path over the top of the barrier (including ground effects where appropriate). The layout is shown in Figure 7.5. Figure 7.5 is drawn using an image source and this allows us to find $\beta$ relatively easily.

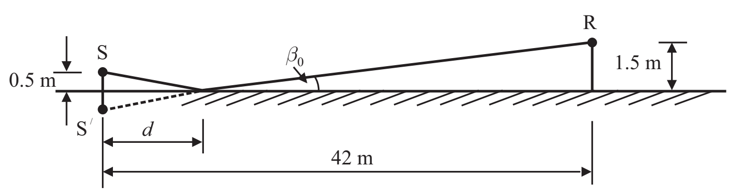


**FIGURE 7.5** Arrangement in absence of barrier for Problem 7.

From similar triangles, the distance, $d_{\mathrm{SR}} = 42/3 = 14$ (m).

Then $\beta_0 = \tan^{-1}(0.5/14) = 2.045°$ and

$$\beta_0\left[\frac{R_1}{\rho f}\right] = \frac{2.045 \times 2 \times 10^5}{1.201 \times 1000} = 339°.$$

Thus, $\frac{\rho f}{R_1} = \frac{1.206 \times 1000}{2 \times 10^5} = 6.03 \times 10^{-3}$

From Figure 4.3 in the textbook, $A_{rf,w} = 5.4$ dB.

Figure 7.6 shows the arrangement with the barrier in place.

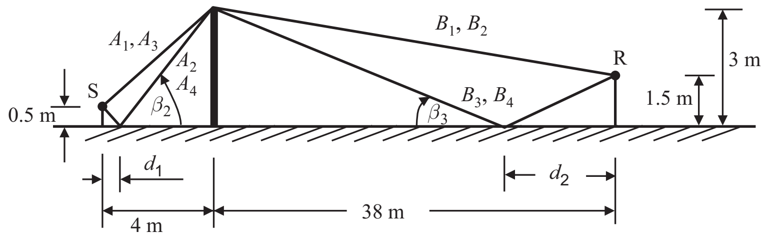


**FIGURE 7.6** Arrangement with barrier in place for Problem 7.

We are only asked to consider the path over the top with no reflections. In this case, we only need to calculate $A_1 + B_1$ and $d_{\text{SR1}}$

$A_1 + B_1 = \sqrt{2.5^2 + 4^2} + \sqrt{1.5^2 + 38^2} = 42.74658$ (m)

Straight line distance, $d_{\text{SR1}} = \sqrt{42^2 + 1^2} = 42.01190$ m

For the path with no ground reflections, Equation (7.38) gives:

$$N_1 = \frac{2 \times 1000}{343}(42.74658 - 42.01190) = 4.28$$

Thus, from Figure 7.19 in the textbook (dashed curve), $\Delta_{b1} = 14.5$ dB.

From Equation (7.39), $A_{b1} = \Delta_{b1} + 20\log_{10}[(A_1 + B_1)/d_{\text{SR1}}]$
$= 14.5 + 20\log_{10}[42.74658/42.01190] = 14.5 + 0.15 = 14.7$ (dB)

From Equation (7.51), the overall barrier Noise reduction (NR $= A_b$) is thus given by:

$$A_b = 10\log_{10}\left[1 + 10^{-(A_{rf,w}/10)} - 10\log_{10}\sum_{i=1}^{n_B} 10^{-(A_{b,i}+A_{rf,i})/10}\right]$$

$$= 10\log_{10}\left[1 + 10^{-5.4/10}\right] - 10\log_{10}\left[10^{-(14.7+0)/10}\right] = 1.1 + 14.7 = 16 \text{ (dB)}$$

SPL at the observer location = 55 - 16 = 39 (dB)

If the problem were done properly, we would take into account the 3 paths involving reflections as well. We will do this now as an exercise and assume that the flow resistance of the ground between the source and receiver is independent of location.

$$\beta_2 = \tan^{-1}\left(\frac{3 + 0.5}{4}\right) = 41.2° \text{ , so}$$

$$\beta_2\left[\frac{R_1}{\rho f}\right] = \frac{41.2 \times 2 \times 10^5}{1.201 \times 1000} = 6830°$$

From Figure 4.3 in the textbook, $A_{rf,2} = 0.2$ dB

$$\beta_3 = \tan^{-1}\left(\frac{3 + 1.5}{38}\right) = 6.75° \text{ , so } \beta_3\left[\frac{R_1}{\rho f}\right] = \frac{6.75 \times 2 \times 10^5}{1.201 \times 1000} = 1120°$$

From Figure 4.3 in the textbook, $A_{rf,3} = 2.6$ dB

Now calculate the Fresnel numbers for the 4 paths over the top:

$A_2 = A_4 = \sqrt{3.5^2 + 4^2} = 5.31507$ m

$B_3 = B_4 = \sqrt{4.5^2 + 38^2} = 38.26552$ m

$A_1 = A_3 = \sqrt{2.5^2 + 4^2} = 4.71700$ m

$B_1 = B_2 = \sqrt{1.5^2 + 38^2} = 38.02959$ m

$A_1 + B_1 = \sqrt{2.5^2 + 4^2} + \sqrt{1.5^2 + 38^2} = 42.74658$ m

Straight line distance, $d_{\text{SR1}} = \sqrt{42^2 + 1^2} = 42.01190$ m

Straight line distance, $d_{\text{SR2}} = \sqrt{42^2 + 2^2} = 42.04759$ m

Straight line distance, $d_{\text{SR3}} = \sqrt{42^2 + 2^2} = 42.04759$ m

Straight line distance, $d_{\text{SR4}} = d_{\text{SR1}} = 42.01190$ m

Path with no ground reflections, $N_1 = \dfrac{2 \times 1000}{343}(42.74658 - 42.01190) = 4.28$

Thus from Figure 7.19 in the textbook, $\Delta_{b1} = 14.5$ dB.

$A_{b1} = \Delta_{b1} + 20\log_{10}[(A_1 + B_1)/d_{\text{SR1}}] = 14.5 + 20\log_{10}[42.74658/42.01190] = 14.5 + 0.15 = 14.7$ dB

Path with ground reflections on source side:

$$N_2 = \frac{2 \times 1000}{343}(5.31507 + 38.02959 - 42.04759) = 7.77$$

Thus, from Figure 7.19 in the textbook, $\Delta_{b2} = 16.6$ dB. $A_{b2} = \Delta_{b2} + 20\log_{10}[(A_2 + B_2)/d_{\text{SR2}}]$

$$= 16.6 + 20\log_{10}[(5.31507 + 38.02959)/42.04759] = 16.6 + 0.13 = 16.7 \text{ (dB)}$$

Path with ground reflections on receiver side:

$$N_3 = \frac{2 \times 1000}{343}(4.71700 + 38.26552 - 42.04759) = 5.66$$

Thus from Figure 7.19 in the textbook, $\Delta_{b3} = 15.5$ dB. $A_{b3} = \Delta_{b3} + 20\log_{10}[(A_3 + B_3)/d_{\text{SR3}}]$

$$= 15.5 + 20\log_{10}[(4.71700 + 38.26552)/42.04759] = 15.5 + 0.2 = 15.7 \text{ (dB)}$$

Path with ground reflections on both sides:

$$N_4 = \frac{2 \times 1000}{343}(5.31507 + 38.26552 - 42.01190) = 9.15$$

Thus from Figure 7.19 in the textbook, $\Delta_{b4} = 17.3$ dB

$$A_{b4} = \Delta_{b4} + 20\log_{10}[(A_4 + B_4)/d_{\text{SR4}}]$$
$$= 17.3 + 20\log_{10}[(5.31507 + 38.26552)/42.01190] = 17.3 + 0.32 = 17.6 \text{ (dB)}$$

Thus:

$$A_b = 10\log_{10}\left[1 + 10^{-(A_{rf,w}/10)} - 10\log_{10}\sum_{i=1}^{n_A} 10^{-(NR_{Bi} + A_{rf,i})/10}\right]$$
$$= 10\log_{10}\left[1 + 10^{-5.4/10}\right]$$
$$- 10\log_{10}\left[10^{-(14.7+0)/10} + 10^{-(17.6+0.2+2.6)/10} + 10^{-(15.7+2.6)/10} + 10^{-(16.7+0.2)/10}\right]$$
$$= 1.1 + 11.1 = 12.2 \text{ dB}$$

SPL at the observer location = 55 - 12 = 43 dB.

# 8

# Solutions to Additional Problems in Chapter 8

## Problem 1

A dissipative silencer contains sound absorbing material whereas a reactive silencer relies on side branches and expansion chambers.

## Problem 2

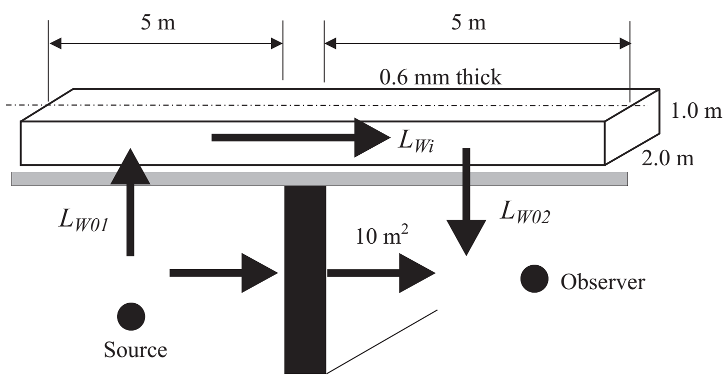


**FIGURE 8.1** Arrangement for Problem 2.

Area of duct on which sound is incident above source room = area of duct radiating into receiver room = $S_d = 5 \times 2 = 10$ (m$^2$)

(a) Difference in sound power level incident on the outside of the air conditioning duct in the source office and the sound power level propagating in one direction inside the duct is given by Equation (8.124) in the textbook as, $L_{Wo1} - L_{Wi} = \text{TL}_\text{i}\text{n} + 3$, where the subscript 1 is used to distinguish the power, $L_{Wo1}$, transmitted into the duct from the power, $L_{Wo2}$, transmitted out of the duct.

(b) The difference in sound power level propagating in one direction inside the duct and the sound power level radiated into the receiving office is given by Equation (8.117) in the textbook as, $L_{Wi} - L_{Wo2} = \text{TL}_\text{o}\text{ut} - 10\log_{10}(P_D L/S) - C$. As there are no losses inside the duct, $C$ of Equation (8.117) is zero.

$10\log_{10}(P_D L/S) = 10\log_{10}(6 \times 5/2) = 11.8$ dB.

$\text{TL}_\text{out}$ is calculated using Equation (8.121) in the textbook for $f < f_{cr}$ and Equation (8.122) for $f_{cr} < f < (f_c/2)$.
$f_{cr} = 612/\sqrt{2} = 433$ Hz.

$$f_c/2 = \frac{0.55c^2}{2c_L h} = \frac{0.55 \times 343^2}{2 \times 6 \times 10^{-4} \times \sqrt{207 \times 10^9/(7850(1-0.3^2))}} \approx 10,000 \text{ Hz}.$$

$\text{TL}_\text{in}$ is calculated using Equation (8.126) in the textbook for $f > f_0$, where $f_0 = c/2a = 343/4 = 86$ Hz.

(c) Twice the ceiling TL is added to the results of items 1 and 2 to obtain $\text{TL}_\text{duct}$ and the results for $\text{TL}_\text{out}$, $\text{TL}_\text{in}$ and $\text{TL}_\text{duct}$ are provided in Table 8.1 below.

(d) The TLs for each 1/3-octave band are combined using:

$$\text{TL}_\text{av} = -10 \times \log_{10}\left[10 \times 10^{-\text{TL}_\text{wall}/10} + 0.04 \times 10^{-\text{TL}_\text{outlet}/10} + 10 \times 10^{-\text{TL}_\text{duct}/10}\right]$$

**TABLE 8.1** TL data and results for Problem 2

| Frequency (Hz) | $\text{TL}_\text{wall}$ | $\text{TL}_\text{ceiling}$ | $\text{TL}_\text{outlet}$ combined source-rec | $\text{TL}_\text{out}$ | $\text{TL}_\text{in}$ | $\text{TL}_\text{duct}$ overall | $\text{TL}_\text{av}$ |
|---|---|---|---|---|---|---|---|
| 100 | 24 | 2 | 15 | 15.6 | 12.6 | 23.5 | 23.7 |
| 125 | 27 | 2 | 15 | 16.6 | 13.6 | 25.4 | 26.0 |
| 160 | 31 | 2 | 15 | 17.7 | 14.7 | 27.6 | 28.8 |
| 200 | 35 | 2 | 15 | 18.6 | 15.6 | 29.5 | 31.1 |
| 250 | 39 | 2 | 16 | 19.6 | 16.6 | 31.5 | 33.3 |
| 315 | 42 | 2 | 16 | 20.6 | 17.6 | 33.5 | 35.1 |
| 400 | 44 | 2 | 17 | 21.7 | 18.7 | 35.5 | 37.0 |
| 500 | 46 | 3 | 17 | 22.4 | 19.4 | 39.0 | 39.4 |
| 630 | 45 | 3 | 18 | 24.4 | 21.4 | 43.0 | 41.4 |
| 800 | 44 | 3 | 19 | 26.5 | 23.5 | 47.2 | 42.6 |
| 1000 | 43 | 3 | 19 | 28.4 | 25.4 | 51.0 | 42.7 |
| 1250 | 44 | 3 | 20 | 30.3 | 27.3 | 54.9 | 43.8 |
| 1600 | 45 | 3 | 20 | 32.5 | 29.5 | 59.2 | 44.4 |
| 2000 | 47 | 3 | 20 | 34.4 | 31.4 | 63.1 | 45.2 |
| 2500 | 49 | 4 | 20 | 36.4 | 33.4 | 69.0 | 45.8 |
| 3150 | 51 | 4 | 20 | 38.4 | 35.4 | 73.0 | 46.2 |
| Overall | | | | | | | 40.4 |

(e) Background + insulation = 35 + 40.4 = 75.4 dBA, so from Table 2.20 in the textbook, the speech privacy rating is on the border line of intelligible.

(f) To increase the speech privacy rating, we could introduce some neutral background noise to raise the ambient to 40 dBA and we could also improve the TL of the wall by bonding a layer of 13 mm thick gypsum board to both sides using a thick silicone adhesive.

**TABLE 8.2** TL data for Problem 2

| Frequency (Hz) | $TL_{wall}$ | $TL_{ceiling}$ | $TL_{outlet}$ | $TL_{duct}$ | Frequency (Hz) | $TL_{wall}$ | $TL_{ceiling}$ | $TL_{outlet}$ | $TL_{duct}$ |
|---|---|---|---|---|---|---|---|---|---|
| 100 | 24 | 10 | 5 | 630 | 45 | 20 | 16 | | |
| 125 | 27 | 12 | 8 | 800 | 44 | 21 | 16 | | |
| 160 | 31 | 14 | 10 | 1000 | 43 | 21 | 16 | | |
| 200 | 35 | 15 | 12 | 1250 | 44 | 21 | 16 | | |
| 250 | 39 | 16 | 12 | 1600 | 45 | 21 | 16 | | |
| 315 | 42 | 18 | 12 | 2000 | 47 | 21 | 16 | | |
| 400 | 44 | 20 | 12 | 2500 | 49 | 21 | 16 | | |
| 500 | 46 | 21 | 14 | 3150 | 51 | 21 | 16 | | |

# Problem 3

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Maximum air flow speed is 7 m/s and required volume flow rate is 0.263 $m^3$ so minimum duct cross sectional area needed is $0.263/7 = 0.0376$ $m^2$. This corresponds to a square section duct of size $0.194 \times 0.194$ (m), so choose $0.2 \times 0.2$ m open section.
Allowed liner thickness $= (0.4 - 0.2)/2 = 100$ (mm), so $\ell/h = 1$
(curve 3 in the liner attenuation figures, 8.32–8.36).
Mach number, $M = 7/343 = 0.02$

Inlet correction (Figure 8.38):
$\lambda = 343/500 = 0.686$ m; $\sqrt{S}/\lambda = 0.2/0.686 = 0.29$; so correction = 5 dB

Exit Loss (Table 8.8):
$D = \sqrt{4 \times 0.04/\pi} = 225$ mm; So loss = 2.5 dB

Required liner loss $= 50 - 7.5 = 42.5$ (dB) or 21.25 dB for each pair of sides.

At 500 Hz, $2h/\lambda = 0.2/0.686 = 0.292$

From Figure 8.32, the maximum attenuation rate for curve 3 for $R_1\ell/\rho c = 2$ is 2.6 dB per $h$ length of duct ($M = 0$).
From Figure 8.33, the maximum rate for curve 3 for $R_1\ell/\rho c = 2$ is 2.2 dB per $h$ length of duct ($M = 0.1$).
Using linear interpolation between the two $M$ values gives, for $M = 0.02$, an attenuation rate of approximately 2.5 dB per $h$ length of duct
so length needed $= 21.25/2.5 = 8.5h$
So choose a 0.9 m or 1 m duct length.

## Problem 4

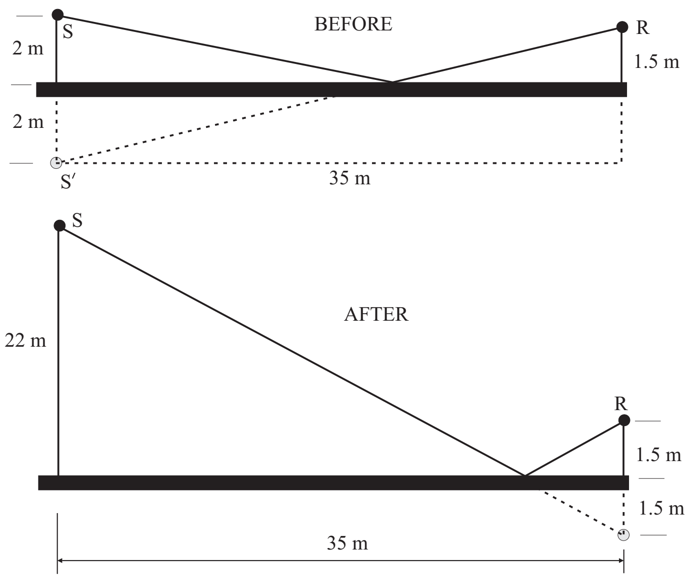


**FIGURE 8.2** Arrangement for Problem 4 before and after installation of the stack.

Noise reduction is the sum of unlined stack losses, directivity changes and spherical spreading. First, calculate spherical spreading reduction.

**BEFORE:**
Direct wave distance $= \sqrt{35^2 + 0.5^2} = 35$ m
Reflected wave distance $= \sqrt{35^2 + 3.5^2} = 35.17$ m
Let direct wave with no stack produce a sound pressure level of $x$ dB at the observer.
Reflected wave then produces a sound pressure level of $x$ dB as well, to give a total $L_p$ of $x{+}3$ dB

**AFTER:**
Direct wave distance $= \sqrt{35^2 + 20.5^2} = 40.56$ m
Reflected wave distance $= \sqrt{35^2 + 23.5^2} = 42.16$ m

$L_p$ for direct wave with stack

$$= x + 20\log_{10}\left[\frac{35}{40.56}\right] = x - 1.28 \text{ dB}$$

$L_p$ for reflected wave with stack

$$= x + 20\log_{10}\left[\frac{35}{42.16}\right] = x - 1.62 \text{ dB}$$

Total $L_p$ with stack
$= 10\log_{10}\left(10^{(x-1.28)/10} + 10^{(x-1.62)/10}\right) = x + 10\log_{10}\left(10^{-0.128} + 10^{-0.162}\right) = x + 1.56$ dB

So reduction in sound pressure level at the receiver due to the increased source receiver distance as a result of the stack raising the height of the noise source is, $x + 3 - (x + 1.56) = 1.4$ dB

Next calculate directivity losses. Before the stack is inserted, angle of observation $= 90°$ . The configuration after installation of stack is shown in Figure 8.3 below.

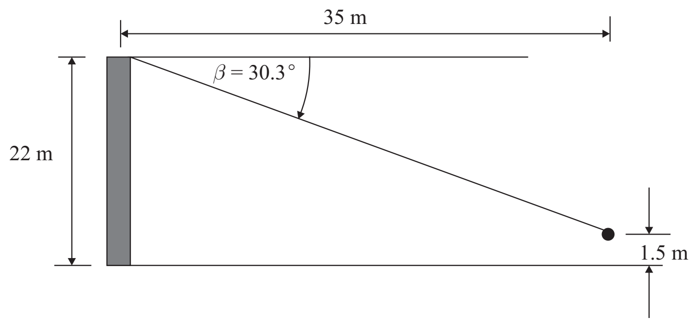


**FIGURE 8.3** Arrangement for Problem 4 after installation of the stack.

8

In this case, $\beta = \tan^{-1}\dfrac{20.5}{35} = 30.3°$; so $\theta = 120°$

$ka = \dfrac{2\pi \times 1000 \times 0.2}{343} = 3.7$

From Figure 8.50 in the textbook, for $ka = 3.7$, the difference in directivity before compared to that after stack installation = 18.5 dB (from 0° to 120° ).

From, Table 8.7, the unlined circular duct attenuation for a 0.4 m diameter stack = 0.18 dB/m, For a stack length of 22 m, this results in an attenuation of approximately 3.6 dB

So total attenuation = 1.4 + 18.5 + 3.6 = 23.5 (dB)

## Problem 5

(a) The normal incidence sound absorption coefficient can be calculated from the standing wave ratio using Equations (5.30) and (5.31). Thus:

$$\alpha_n = 1 - \left[\frac{10^{L_0/20} - 1}{10^{L_0/20} + 1}\right]^2 = 1 - \left[\frac{10^{15/20} - 1}{10^{15/20} + 1}\right]^2 = 0.513$$

(b) The effective length of the holes in the perforated sheet given by Equations (8.7) and (8.12) in the textbook. The distance between holes is calculated using the equations in the solution to problem 5.9 in the textbook. For parallel holes:

$$q = \sqrt{\frac{\pi \times d_h^2}{4 \times (P/100)}} = \sqrt{\frac{\pi \times 0.002^2}{4 \times (0.1)}} = 5.6 \text{ mm.}$$

For staggered holes:

$$q = \sqrt{\frac{\pi \times d_h^2}{2 \times (P/100)\sqrt{3}}} = \sqrt{\frac{\pi \times 0.002^2}{2 \times 0.1\sqrt{3}}} = 6.0 \text{ mm.}$$

Alternatively, for the parallel configuration, $q$ can be calculated by first calculating the number of holes in a square metre of panel. For a hole diameter of 2 mm and an open area of 10%, the number of holes per m$^2$ is:

$$N = \frac{0.1 \times 4}{\pi \times 0.002^2} = 31831$$

The hole spacing is then the square root of (panel area divided by number of holes):

Thus, $q = \sqrt{\dfrac{1000^2}{31831}} = 5.6$ (mm). We shall use this configuration.

The hole effective length is then obtained using Equations (8.7) and (8.12) in the textbook, assuming that the effect of the 0.4 m diameter tube on the perforated sheet hole effective length is negligible. Thus:

$$\ell_e = 0.0016 + \frac{16 \times 0.001}{3\pi}\left(1 - \frac{0.43 \times 0.001}{0.0056}\right)\left(1 - 0.15^2\right) = 0.0031 \text{ m}$$

(c) From Equation (1.13), $k = \frac{2\pi f}{c} = \frac{2\pi \times 300}{343} = 5.495 \text{ m}^{-1}$

$k\ell_e = 0.017$, so $\tan k\ell_e \approx k\ell_e$;

$P = 10\%$; Cross-sectional area of the duct, $S = \frac{\pi \times 0.4^2}{4} = 0.1257 \text{ m}^2$

$\rho c = 413.6$ (kg m$^{-2}$ s$^{-1}$), $M = 0.15$, which has the same effect as a flow from left to right in the same direction as sound propagation (see discussion in Section 8.7.1);

so $\tan[k\ell_e(1 - M)] \approx k\ell_e(1 - M)$;

$m = 7850 \times 0.0016 \times 0.9 = 11.30 \text{ kg/m}^2$

From Equation (8.16), $Z_A = \dfrac{\frac{100}{PS}\left[\mathrm{j}\rho c\left(k\ell_e(1-M)\right) + 0.7\rho c M\right]}{1 + \frac{100}{\mathrm{j}\omega m P}\left(\mathrm{j}\rho c k\ell_e(1-M) + 0.7\rho c M\right)}$

$$= \frac{100}{10 \times 0.1257}\left[\frac{413.6\mathrm{j} \times 0.017 \times 0.85 + 413.6 \times 0.7 \times 0.15}{1 - \frac{100\mathrm{j}}{2\pi \times 300 \times 11.30 \times 10}\left(\mathrm{j} \times 413.6 \times 0.017 \times 0.85 + 413.6 \times 0.7 \times 0.15\right)}\right]$$

$= 3430 + 550\mathrm{j}$ kg m$^{-4}$ s$^{-1}$ or Rayls/m$^2$

If we ignored the panel mass contribution, we would remove the denominator in the square brackets from the above expression to give:

$Z_A = 3460 + 482\mathrm{j}$.

## Problem 6

(a) Equivalent electrical circuit.

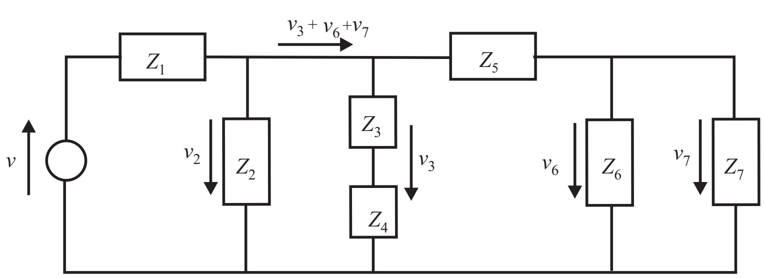


**FIGURE 8.4** Equivalent electrical circuit for Problem 7.

(b) $v = v_2 + v_3 + v_6 + v_7$

$v_2 Z_2 = v_3(Z_3 + Z_4) = (v_6 + v_7)Z_5 + v_6 Z_6$

$v_6 = \dfrac{v_7 Z_7}{Z_6}$

Thus, $v_2 = \dfrac{Z_5 v_6}{Z_2} + \dfrac{Z_5 v_7}{Z_2} + \dfrac{v_6 Z_6}{Z_2}$

$$v_3 = \frac{Z_5 v_6}{Z_3 + Z_4} + \frac{Z_5 v_7}{Z_3 + Z_4} + \frac{v_6 Z_6}{Z_3 + Z_4}$$
$$v_2 = v_7 \left[ \frac{Z_5 Z_7}{Z_2 Z_6} + \frac{Z_5}{Z_2} + \frac{Z_7}{Z_2} \right]$$
$$v_3 = v_7 \left[ \frac{Z_5 Z_7}{(Z_3 + Z_4) Z_6} + \frac{Z_5}{(Z_3 + Z_4)} + \frac{Z_7}{(Z_3 + Z_4)} \right]$$
$$\frac{v}{v_7} = \left[ \frac{Z_5 Z_7}{Z_2 Z_6} + \frac{Z_5}{Z_2} + \frac{Z_7}{Z_2} + \frac{Z_5 Z_7}{(Z_3 + Z_4) Z_6} + \frac{Z_5}{(Z_3 + Z_4)} + \frac{Z_7}{(Z_3 + Z_4)} + \frac{Z_7}{Z_6} + 1 \right]$$

Insertion Loss is then $\mathrm{IL} = 20 \log_{10} \left[ \frac{v}{v_7} \right]$

## Problem 7

Dissipative muffler - 3 attenuations: inlet, outlet and lined section (no expansion loss as it is mounted on an enclosure). Try beginning with the smallest allowed cross section of duct. Thus: $S = 0.25$ m$^2$, $\sqrt{S}/\lambda = f \times \sqrt{S}/c = 1.458 \times 10^{-3} f$ and $2h/\lambda = \sqrt{0.25} f/c = 1.458 \times 10^{-3} f$. The inlet loss is obtained from Figure 8.38 and the outlet loss is obtained from the numbers in brackets in Table 8.8. The equivalent circular duct diameter for use in Table 8.8 is $D = \sqrt{4S/\pi} = 0.318$ m.

**TABLE 8.3** Requirements for Problem 8

| frequency (Hz) | $\sqrt{\frac{S}{\lambda}} = \frac{2h}{\lambda}$ | inlet loss (dB) | outlet loss (dB) | total atten. needed (dB) | Liner atten. needed (dB) |
|---|---|---|---|---|---|
| 125 | 0.18 | 2 | 9 | 9 | 0 |
| 1000 | 1.46 | 10 | 1 | 15 | 4 |
| 2000 | 2.91 | 10 | 0 | 15 | 5 |

If we use the largest outer cross section allowed, the ratio of liner thickness to half airway width is $\ell/h = 1.0$. Using curve 3 in Figure 8.33 in the textbook, for a flow speed of M = 0.1, the best flow resistance choice is $R_1 \ell/(\rho c) = 16$ and this gives following attenuations for a $h = 0.25$ m length of duct lined on all 4 sides.

**TABLE 8.4** Results for Problem 8

| 1/3 octave centre frequency (Hz) | IL (dB) |
|---|---|
| 125 | $0.7 \times 2$ |
| 1000 | $2.2 \times 2$ |
| 2000 | $0.55 \times 2$ |

It is clear that the important frequency is 2000 Hz and to satisfy the requirement of 5 dB at this frequency, we need a length of duct equal to $5 \times 0.25/1.1 = 1.14$ m. Use 1.2 m.

## Problem 8

(a) .

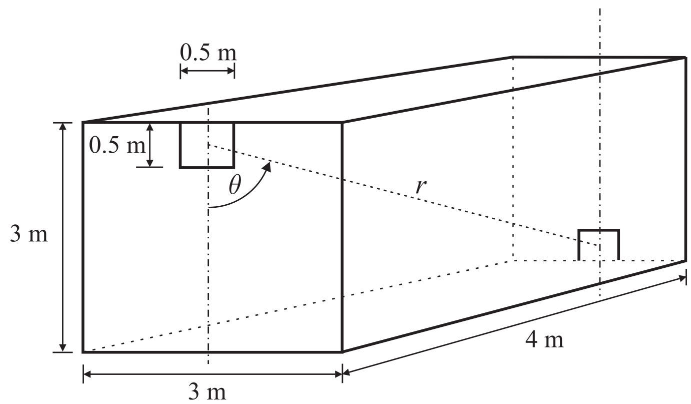


**FIGURE 8.5** Arrangement for Problem 9.

(b) $r = \sqrt{3 - 0.25 - 0.25)2 + 4^2} = 4.717$ m
$\cos\theta = 4/4.72 = 0.848, \quad \theta = 32°$
Wall area, $S = 2 \times (3 \times 3 + 4 \times 3 \times 2) = 66$ m$^2$
Outlet area, $A_{\text{exit}} = 0.5 \times 0.5 = 0.25$ m$^2$
$\alpha = 0.2$

Inlet duct cut-on frequency is $f_{co} = 343/(2 \times 0.5) = 343$ Hz.

**Wells Method** (Equation (8.131))

$$\text{TL} = -10\log_{10}\left[\frac{0.5 \times 0.5 \times 0.8}{66 \times 0.2} + \frac{0.5 \times 0.5 \times 0.848}{\pi \times 4.72^2}\right] = 17.4 \text{ dB}$$

**ASHRAE method** (Equation (8.132))

$$\text{As } f_{co} < 630, \text{ TL} = 3.505\left[\frac{0.5 \times 0.5 \times 0.8}{66 \times 0.2} + \frac{0.5 \times 0.5}{\pi \times 4.72^2}\right] + 3.0 = 17.6 \text{ dB}$$

(c) Increasing $\alpha$ to 0.5 requires changing 0.2 to 0.5 in the above equations, resulting in TL = 21.7 dB for the Wells method and TL = 23.4 dB for the ASHRAE method.

(d) The approximate way of solving this problem is to remove the direct field component from the equations for the plenum chamber TL. This is the first term in brackets in Equations (8.131) and (8.132). When this is done the increase in TL for the Wells method is 0.8 dB and for the ASHRAE method, it is 1.2 dB.

However, the additional noise reduction due to the partition can also be found by treating the problem as an indoor partition problem and Equation, (7.38) and (7.55). In this case, we assume that the partition is covered with sound absorbing material so that the average Sabine sound absorption coefficient for the plenum spaces on each side of the partition is the same as the average Sabine sound absorption coefficient for the plenum with no central partition. Referring to Equations (7.38) and (7.55):
$S_0 = 66$ m$^2$
$\alpha_0 = 0.2$
$r = d = 4.717$
$S = 3 \times 0.2 = 0.6$ m$^2$
$A = \sqrt{2.75^2 + 2^2} = 3.40$ m

$B = \sqrt{0.25^2 + 2^2} = 2.016$ m
Fresnel number, $N = (2 \times 343/630) * (3.400 + 2.016 - 4.717) = 2.568$
$F = 1/(3 + 10 \times 2.568) = 0.0349$
$S_1 = S_2 = 0.5 \times 66 + 3 \times 2.8 = 41.4$ m$^2$
$K_1 = K_2 = 0.6/(0.6 + 41.4 \times 0.2) = 0.0676$

Additional IL due to partition is $\text{IL} = 10\log_{10}\left(\frac{1}{2\pi \times 4.717^2} + \frac{4}{66 \times 0.2}\right)$

$$- 10\log_{10}\left(\frac{0.0349}{2\pi \times 4.717^2} + \frac{4 \times 0.0676 \times 0.0676}{0.6 \times (1 - 0.0676 \times 0.0676}\right) = 10.0 \text{ dB}$$

A possible reason for the large difference between this result and the approximate result is that the approximate method does not allow for the reduction in reverberant sound pressure level on the exit side of the central partition.

(e) The sound pressure level in the downstream duct is equal to the sound pressure level in the upstream duct minus the TL of the plenum chamber. According to the Wells method, this would equal $90 - 17.4 = 72.6$ dB and for the ASHRAE method the estimated sound pressure level in the downstream duct would be $90 - 17.6 = 72.4$ dB.